Lingfield Education Trust

Calculation Guidance

Contents

- This contents list is hyperlinked for ease of access
- The orange dots or Lingfield logo on each page are then hyperlinked back to this contents page

Background, Purpose and Aims

Mathematics has learning episodes that can be taught in multiple different ways, using multiple different representations and methods; **this can cause significant confusion and cognitive overload for some students, especially lower attaining students.**

The purpose of this document is to provide teachers and staff, who support students in mathematics lessons at Lingfield Education Trust, with an easy-reference guide to the methods that could be employed in the teaching of mathematics.
The key principles underpinning this guidance are:

- The importance of mental calculation methods, that are themselves built on secure factual knowledge.
- Giving pupils in each year group a reliable method for calculating that they can apply to varied representations, reasoning and problem-solving. Although there is minimal reference to bar modelling and part-part-whole models in the document, pupils should still be exposed to them regularly through your maths curriculum – this document is for the strategies you would use to complete the missing numbers in both of the aforementioned models.
- Reducing the amount of variation pupils are exposed to in the initial learning phase of calculating in a given year group. Variation is essential to a deep understanding, however we understand that a firm foundation is needed first.
- The importance of the concrete, pictorial and abstract phases of learning.
- Using the right manipulative at the right time – if it is needed.
- Building on prior learning through the careful sequencing of strategies.

The aim of this guidance is to allow staff to synchronise their practise, to ensure students encounter the same methods throughout their mathematical journey, regardless of their teacher. The aim is that this will provide consistency for students in the long-term and therefore aid in improving their outcomes.

This document was created by members of Lingfield Education Trust's Maths Network based on their teaching expertise, the most up-to-date research and through the study of effective exemplars.

Concrete, Pictorial and Abstract

Throughout this document each approach is split into three stages: concrete, pictorial and abstract. The idea is through a systematic approach students will begin, where possible, to explore mathematics by using physical manipulatives so that at the end of the process students should be able to form their own generalisations of mathematical rules.

Concrete	During the concrete stage, pupils will have the opportunity to work with manipulatives and other physical objects in order to understand the mathematical concept. There will be times where this is not possible or effective; in these cases students should begin at pictorial stage.
Pictorial	During the pictorial stage, pupils should be able to pictorially or diagrammatically represent ideas discovered during the Physical Stage. Again there may be occasions where this is not effective and so pupils should start at the abstract stage.
Abstract	During the abstract stage, pupils should no longer require a diagram to understand the concept. They should have formed comprehensive generalisations during which the underlying mathematics is fully understood.

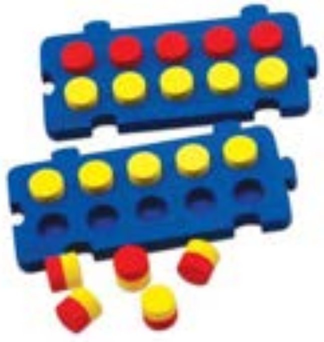
Mathematical Manipulatives

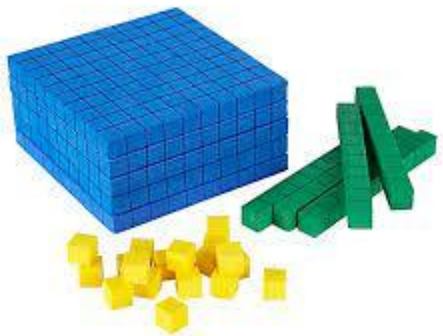
In order for our children to fully understand the structures within calculation, we use a set of key maths manipulatives to **expose** the maths.

We expect all of our children to progress to **doing** the maths **without** these resources, when secure.

There is a clear rationale for when each manipulative is introduced:

- **Numicon:** this resource helps early years children see numbers inside other numbers and begin to develop fact fluency.
- **Five/Ten Frames for calculations to 20:** this resource helps expose the concept of 10 and how to bridge it.
- **Base 10 for calculations to 100:** this resources helps to expose the concepts of regrouping and exchanging; it is also less cumbersome than having multiple tens frames on every desk.
- **Place Value Counters for calculations beyond 100:** as Base 10 can be expensive and space-taking for every child to have equipment, we switch to place value counters at this point.

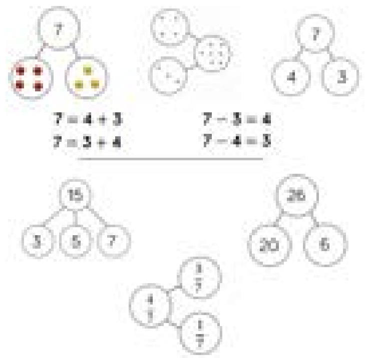




Models & Structures

This document aims to outline the main calculation strategies to be used progressively across school. There are however a range of models and representations that help pupils draw out the structure of the maths behind a task/question – in other words help pupils identify the operation and arithmetic required. This page details some of the most effective that you should use to help pupils expose the structure of the maths before they apply a mental or written strategy to complete the calculation(s).

Part-Part-Whole Model



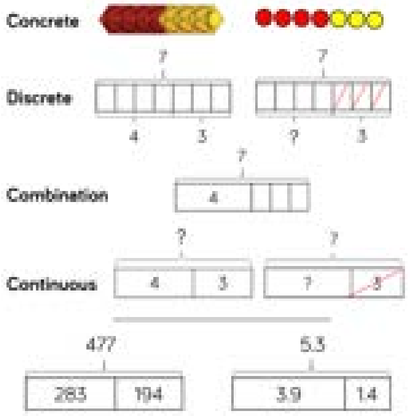
This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part. Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (Single)

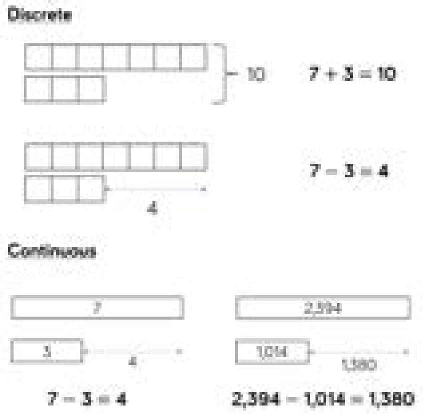


The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure. Cubes and counters can be used in a line as a concrete representation of the bar model. Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model. Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (Multiple)



The multiple bar model is a good way to compare quantities whilst still unpicking the structure. Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

The Language of Calculation

Mathematics is a language, and just like any other language, it is important for students to have a strong vocabulary in order to be successful. As a trust we encourage the teaching, practice and application of precise mathematical vocabulary for several reasons:

- Allows children to reason about maths accurately
- Provides equity with other pupils who know such vocabulary
- Builds whole school consistency to lower cognitive load (if one teacher uses one word and another a different one 'brain power' is diverted from the actual maths at hand.
- In the case of calculation, knowing the correct terminology often allows children to work out whether the whole is needed (the answer) or one of the parts (missing number).

The images below show the key vocabulary to be taught alongside the calculations in this guidance document.

Precise Mathematical Vocabulary

Addition

addends: the quantities being added
sum: the total when two or more addends are added together

Subtraction

minuend: the quantity being subtracted from
subtrahend: the amount being taken
difference: what is left after a subtrahend has been taken from a minuend

Multiplication

factors: the two amounts being multiplied
multiplier: the amount being multiplied by
multiplicand: the amount being multiplied
product: the answer when two factors are multiplied / the answer when a multiplicand is multiplied by a multiplier

Division

dividend: the amount being divided
divisor: the amount being divided by
quotient: the answer when a dividend is divided by a divisor

addend + addend = sum

minuend - subtrahend = difference

multiplier x multiplicand = product

dividend ÷ divisor = quotient

Factual Knowledge

The written calculation strategies contained in this document are built up on the mental calculation strategies outlined on the next page, however these themselves are built on secure factual knowledge (fact fluency). These are the key milestones in what **factual knowledge** pupils should know to automaticity and by when. This does not just mean rote learning but using strategies to develop understanding through to automaticity. Programs to use for this are one of: Number Sense, NCETM Mastering Number or WR Fluency Bee.

EYFS

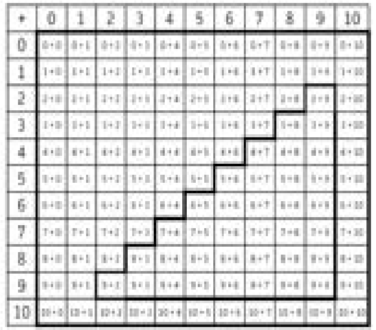
Have a deep understanding of number to 10; including the composition of each number.

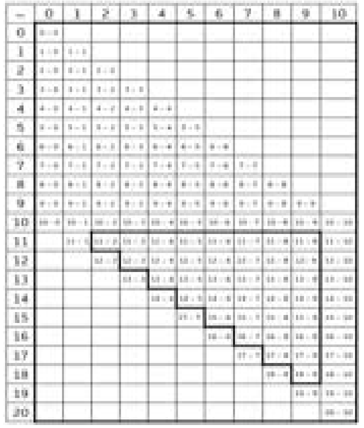
Subitise to 5.

Automatically recall (without reference to rhymes or other aides) number bonds to 5 (including subtraction facts) and some to 10 including doubles.

Year 1

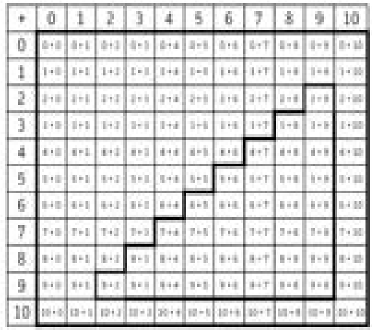
All addition and subtraction facts to 20 that do not bridge 10. They will be exposed to bridging 10 but automaticity is not required.

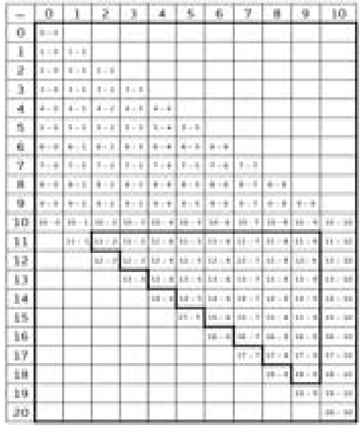




Year 2

All addition and subtraction facts to 20 including those that bridge 10.





Year 4

All multiplication and division facts to 12 x 12.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Mental Calculation Expectations

	Addition	Subtraction	Multiplication	Division
YR	• Perceptually subitise to 10 • Conceptually subitise to 5 • Find the total number of items in two groups, up to a total of 10 *(combine and subitise, count all (aggregation), use known facts)* • 1 more to 10 • Add zero, within numbers to 10	• 1 less to 10 • Remove from a small group and find how many are left, up to a total of 10 *(take away and subitise, take away and count how many are left, use known facts)* • Subtract zero to 10	• Doubles to 5	
Year 1	• Subitising 1-5 • Recognizing numbers on tens frames • Add 1-digit to tens • Add 1-digit to teens • Number Bonds to 10 • Bridging 10 single digits • Near doubles to 5, e.g. 3+2	• Subtract pairs of 1-digit numbers • Subtraction facts to 10 • Bridging 10 by single digit subtraction • Subtract1-digit from teens • Subtract1-digit from ten •	• Double numbers to 5 • Count forwards and backwards in 2s, 5s and 10s	• Halve even numbers to 10
Year 2	• Bridging 10 (TU + U) • 1-digit to a multiple of ten (e.g. 60 + 5) • Add multiples of 10 to a 2-digit number (e.g. 27 + 60) • Add three 1-digit numbers • Number Bonds to 20 • Number Bonds to 100 in 10s • Add 10 to 2-digit numbers using place value • Add 11 by adding 10 add 1 • Add 9 by add 10 take 1 • Near doubles to 10, e.g 6+5	• Subtract 10 from a 2-digit number using place value • Bridging any 2-digit10 by single digit subtraction • Subtract 1-digit from multiple of 10 • Subtraction facts to 20 • Subtraction facts to 100 in 10s • Subtract 11 by subtracting 10 then 1 • Subtract 9 by subtracting 10 and adding 1	• Double numbers to 10 • Double any multiple of 10 up to 50 • Recognize odd and even • Rapid recall of x2,10,5 as a minimum	• Halve even numbers to 20 • Halve any multiple of 10 with an even tens digit up to 100 • Rapid recall of division facts for x2,10,5 as a minimum
Year 3	• Add 100 to any 3-digit number using place value • Bridging to 3-digit • Add pairs of multiples of 10 up to 2-digit using bonds • 2-digit Near Doubles (teens and tens, e.g. 14 + 13, 30 + 20) • 2-digit near 10s round up (e.g. 27 + 19/21) • Add any 2-digit numbers using partitioning • Add any 2-digit numbers using counting on	• Subtract 100 from any 3-digit number using place value • Bridging HTU by U subtraction • Subtract a 2-digit number from a multiple of 10 • Subtract pairs of multiples of 10 up to 2-digit using bonds • Subtract near multiples of 10 rounding up • Subtract pairs of 2-digit using partitioning • Subtract pairs of 2-digit using counting on	• Double any multiple of 10 up to 100 • Find 4 of a number by doubling and doubling again • Rapid recall of x3, 4,8 as a minimum • Multiply any 2-digit number by 10 • Multiply TU x U using partitioning • Use place value and known facts to TU x U, e.g. 80 x 3	• Halve any multiple of 10 up to 100 • Find a quarter by halving and halving again • Rapid recall of division facts for x3,4,8 as a minimum • Identify the remainder when dividing TU by 2,10,5 • Divide any 3-digit multiple of 10 by 10 • Use place value and known facts to HTU ÷ U, e.g. 400 ÷ 8

Experience has shown us that longer, more complex written methods often go wrong through the **mental calculations** within them.

It is essential that pupils are taught these mental calculation skills.

Once pupils have mastered the relevant mental and written methods for their year group, it is advisable for them to **reason about which method suits a given calculation – what was the most efficient way of doing it!**

Please see our mental calculation policy for further detail to support these expectations.

Mental Calculation Expectations

Year 4	• Add 1000 to any 4-digit number using place value • Bridging up to 4-digit • Add pairs of multiples of 10 up to 3-digit using bonds • 2-digit Near Doubles to 50, e.g. 36 + 37 • 2-digit near 10s round up & down (e.g. 27 + 19/21) • Add any 3-digit numbers using partitioning • Add any 3-digit numbers using counting on	• Subtract 1000 from any 4-digit number using place value • Bridging THTU by U subtraction • Subtract pairs of multiples of 10 up to 3-digit using bonds • Subtract near multiples of 10 rounding up and down • Subtract any 3-digit numbers using partitioning • Subtract any 3-digit numbers using counting on •	• Double any 2-digit number • Double any multiple of 100 • Rapid recall of all tables to 12x12 • Multiply three 1-digit numbers • Multiply any number to 100 by 10/100 • Multiply HTU x U using partitioning • Use place value and known facts to HTU x U, e.g. 400 x 3	• Halve any even number to 100 • Rapid recall of all division facts for tables to 12x12 • Identify the remainder when dividing HTU by 2,10,5 • Divide any number to 1000 by 10/100 • Use place value and known facts to THTU ÷ U, e.g. 1200 ÷ 3
Year 5	• Use place value to add powers of 10 to 1,000,000 • Bridging (U.t + .t) • 2-digit Near Doubles to 100, e.g. 76 + 77 • Add near hundreds (e.g. 427 + 198) • Add any U.t pairs (e.g 3.5 + 2.8) using partitioning • Add any U.t pairs (e.g 3.5 + 2.8) using counting on • Add pairs of multiples of U.t by making x10 larger	• Use place value to subtract powers of 10 up to 1,000,000 • Bridging U.t by U subtraction • Subtract near hundreds (e.g. 427 - 198) • subtract any U.t pairs (e.g 3.5 - 2.2) using partitioning • subtract any U.t pairs (e.g 3.5 - 2.7) using counting on • Subtract pairs of multiples of U.t by making x10 larger	• Double 3-digit multiples of 10 • Double U.t • Multiply whole numbers by 10,100,1000 • Multiply U.t using partitioning • Use place value and known facts to THTU x U, e.g. 8000 x 3 • Multiply pairs of multiples of 10 with same place value ,e.g. 400 x 300 • Multiply by 50 by multiplying by 100 and halving • Multiply by 25 by multiplying by 100 and halving and halving again • Multiply by 20 by multiplying by 10 and doubling • Multiply by 5 by multiplying by 10 and halving	• Halve 3-digit multiples of 10 • Halve any whole number • Find the remainder when dividing TU by any single digit • Divide whole numbers by 10,100,1000 • Use place value and known facts to TTHTU ÷ U, e.g. 64000 ÷ 8 • Multiply pairs of multiples of 10 with same place value, e.g. 800 ÷ 200
Year 6	• Use place value to add powers of 10 to any number • Bridging (U.th + .th) • Near doubles to tenths (e.g. 1.7 + 1.6) • Near tens to tenths (e.g. 4.2 + 1.9) • Add any U.th pairs (e.g 3.52 + 2.87) using partitioning • Add any U.th pairs (e.g 3.52 + 2.87) counting on	• Use place value to subtract powers of 10 from any number • Subtract using near tens to tenths, e.g. 4.6 – 1.9 • Subtract any U.th pairs (e.g 3.52 - 2.31) using partitioning • Subtract any U.th pairs (e.g 3.52 - 2.31) using counting on	• Double any number including to 2dp • Multiply whole numbers and decimals by 10,100,1000 • Multiply U.th x U using partitioning • Use place value and known facts for decimals, e.g. 0.3 x 4 • Multiply pairs of multiples of 10 with differing place value ,e.g. 4000 x 30	• Halve any number including 2dp • Divide whole numbers and decimals by 10,100,1000 • Use place value and known facts for decimals, e.g. 3.2 ÷ 8 • Divide pairs of multiples of 10 with differing place value, e.g. 8000 ÷ 200 • Divide by 50 by dividing by 100 and doubling • Divide by 25 by dividing by 100 and doubling and doubling again • Divide by 20 by dividing by 10 and halving • Divide by 5 by diving by 10 and doubling

Experience has shown us that longer, more complex written methods often go wrong through the **mental calculations** within them.

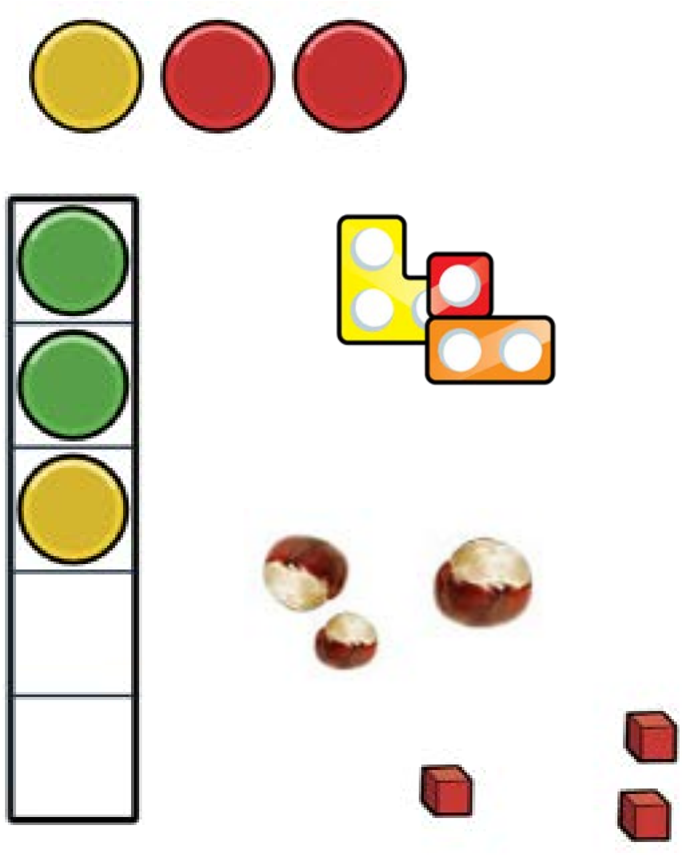
It is essential that pupils are taught these mental calculation skills.

Once pupils have mastered the relevant mental and written methods for their year group, it is advisable for them to **reason about which method suits a given calculation – what was the most efficient way of doing it!**

Please see our mental calculation policy for further detail to support these expectations.

Addition

Nursery

Concrete	Pictorial	Abstract
Pupils to use a range of practical resources to add numbers up to three. Ensure this includes numicon overlays for numbers within numbers. 	All addition work will fall within the concrete phase with practical resources at this age.	All addition work will fall within the concrete phase with practical resources at this age.

There is to be **no** reference to equations in EYFS – neither verbal or written.

ELG
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Reception

Concrete

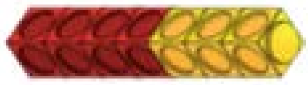
Pupils to use a range of practical resources to understand number composition to ten. Ensure this includes numicon overlays for numbers within numbers.

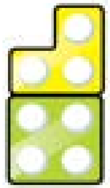
Pupils to use practical resources to understands parts & wholes and numbers inside other numbers for addition facts.

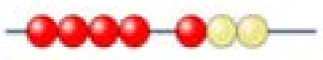
Four and three are parts of seven.

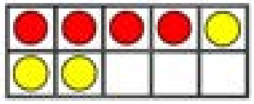
Four is a part; three is a part; seven is the whole.

4 and 3 are numbers inside 7.



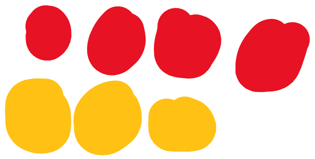


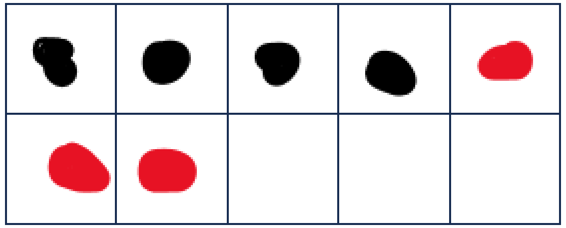


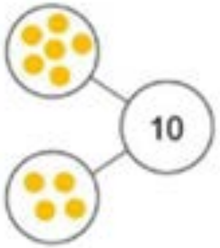


Pictorial

Children represent concrete resources & strategies pictorially.







Abstract

All addition work will fall within the concrete and pictorial phases with practical resources at this age.

There is to be **no** reference to equations in EYFS – neither verbal or written.

National Curriculum
add one-digit and two-digit numbers to 20, including zero; read, write and interpret mathematical statements involving addition (+) and equals (=) signs

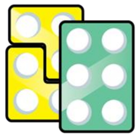
Year 1 (to and within 10)

Concrete

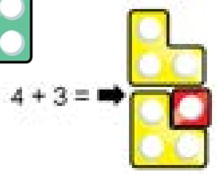
Children use a range of concrete resources – **not** including graded number lines – to practice the following addition strategies for facts to and within 10.

- Adding zero / adding to zero
- 1 more
- 2 more

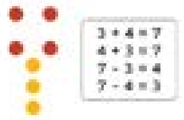
- Doubles



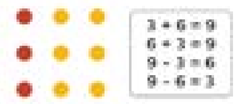
- Near doubles



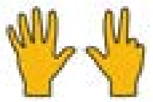
- 7-Tree

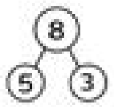


9-Square



- Five-and-a-bit





Pictorial

Children represent concrete resources & strategies pictorially.

Abstract

Pupils to record their addition calculations as mathematical statements (number sentences) using the addition and subtraction symbols.

$$4 + 3 = 7$$

There is to be **no** use of graded number lines for addition.

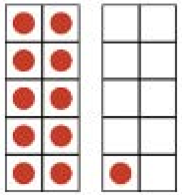
National Curriculum
add one-digit and two-digit numbers to 20, including zero; read, write and interpret mathematical statements involving addition (+) and equals (=) signs

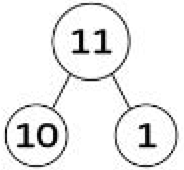
Year 1 (10 to 20)

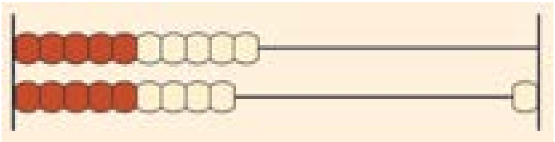
Concrete

Children use a range of concrete resources – **not** including graded number lines – to practice the following addition strategies for facts from 10 to 20 (no bridging 10).

- Ten and a bit







Pictorial

Children represent concrete resources & strategies pictorially.

Abstract

Pupils to record their addition calculations as mathematical statements (number sentences) using the subtraction symbol.

$$14 + 3 = 17$$

There is to be **no** use of graded number lines for addition.

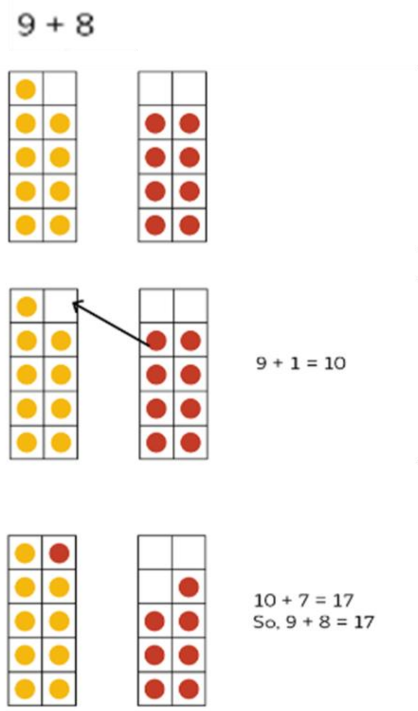
National Curriculum
add one-digit and two-digit numbers to 20, including zero; read, write and interpret mathematical statements involving addition (+) and equals (=) signs

Year 2 (bridging 10)

Concrete

Children use a range of concrete resources – **not** including graded number lines – to practice the following addition strategies for bridging ten.

- Make ten and then

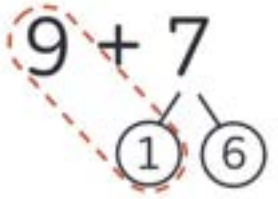


Pictorial

Children represent concrete resources & strategies pictorially.

Abstract

Pupils to use make ten and then in written form.



So,
9 + 7 = 9 + 1 + 6
= 16

There is to be **no** use of graded number lines for addition.

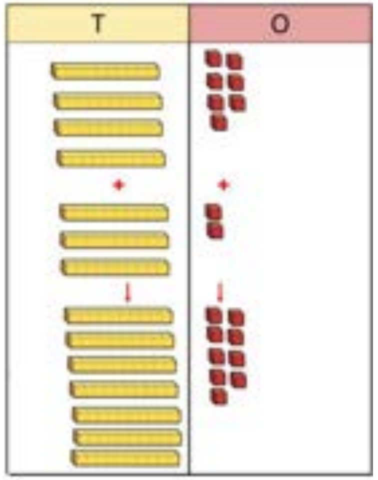
National Curriculum
add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers adding three one-digit numbers

Year 2

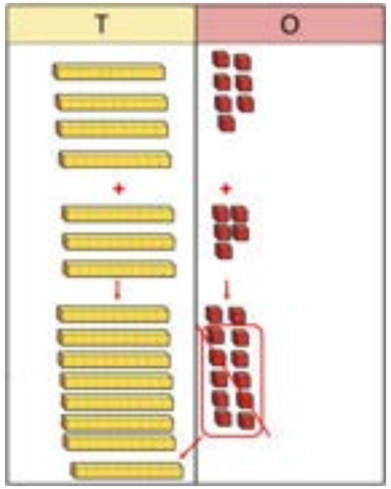
Concrete

Pupils to use Base 10 to practically experience adding and regrouping. This must also be done with counters ready for Year 3.

47 + 32 = 79



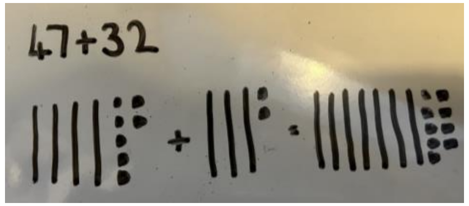
47 + 35 = 82



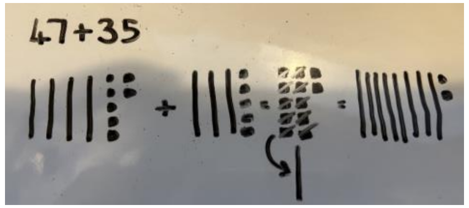
Pictorial

Pupils to draw Base 10 images, which again must move into using counters ready for Year 3.

47 + 32 = 79



47 + 35 = 82



Abstract

Pupils to use expanded method with no regrouping before moving onto it.

47 + 32 = 79

	40	+	7		
+	30	+	2		
	70	+	9	=	79

47 + 35 = 82

	40	+	7		
+	30	+	5		
	70	+	12	=	82

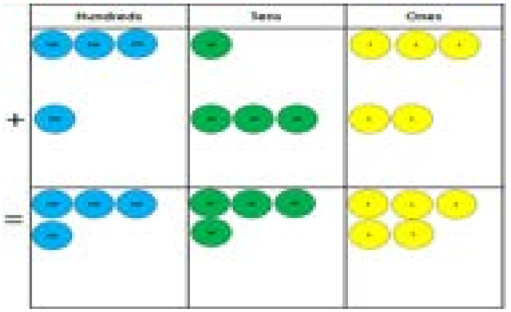
National Curriculum
add numbers with up to three digits, using formal written methods of columnar addition

Year 3

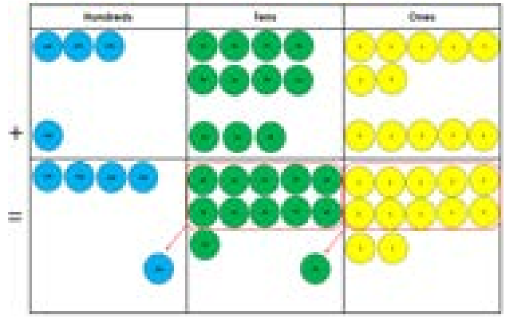
Concrete

Pupils to use counters (or **Base** 10)to practically experience adding.

313 + 132 = 445



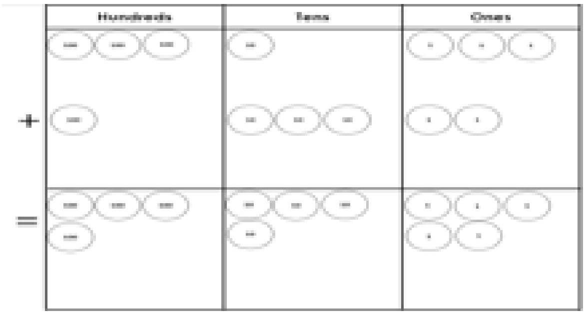
387 + 135 = 522



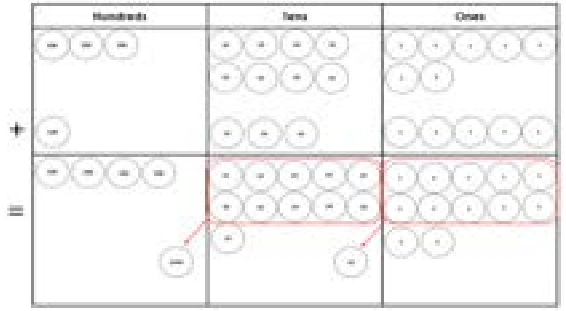
Pictorial

Pupils to draw counters without crossing out for regrouping.

313 + 132 = 445



387 + 135 = 522



Abstract

Pupils to use expanded method with no regrouping before moving onto it.

$$\begin{array}{rcrcr} 300 & + & 10 & + & 3 \\ -\ 100 & + & 30 & + & 2 \\ \hline 400 & & 40 & + & 5 \\ \hline \end{array} \quad = \quad 445$$

$$\begin{array}{rcrcr} 300 & + & 80 & + & 7 \\ +\ 100 & + & 30 & + & 5 \\ \hline 400 & + & 110 & + & 12 \\ \hline \end{array} \quad = \quad 522$$

Pupils to move onto trying compact, standard method ready for Year 4 when secure with the above. **CPA not need between the two abstract models;** ensure both abstract models are related to each other in teaching.

$$\begin{array}{rccc} & 2 & 3 & 1 \\ + & 3 & 2 & 2 \\ \hline & 5 & 5 & 3 \\ \hline \end{array}$$

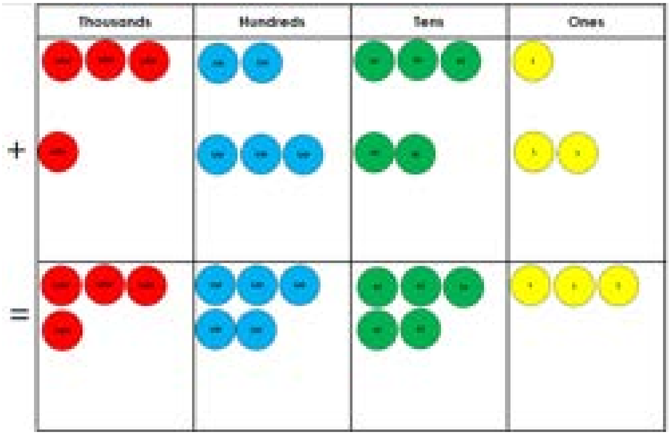
National Curriculum
add numbers with up to 4 digits using the formal written method of columnar addition where appropriate

Year 4

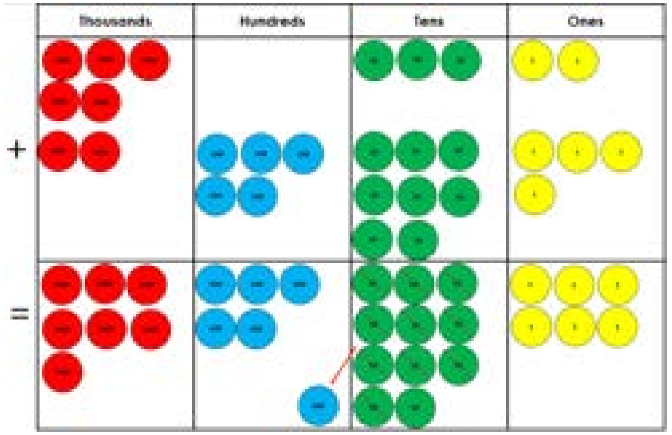
Concrete

Pupils to use counters to practically experience adding.

3231 + 1322 = 4553



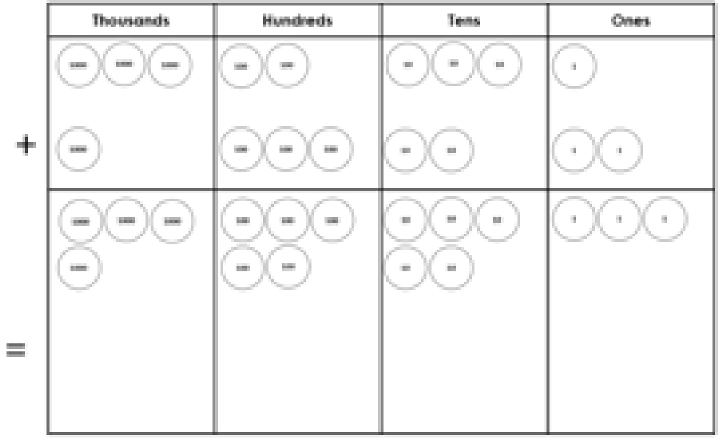
5032 + 2584 = 7616



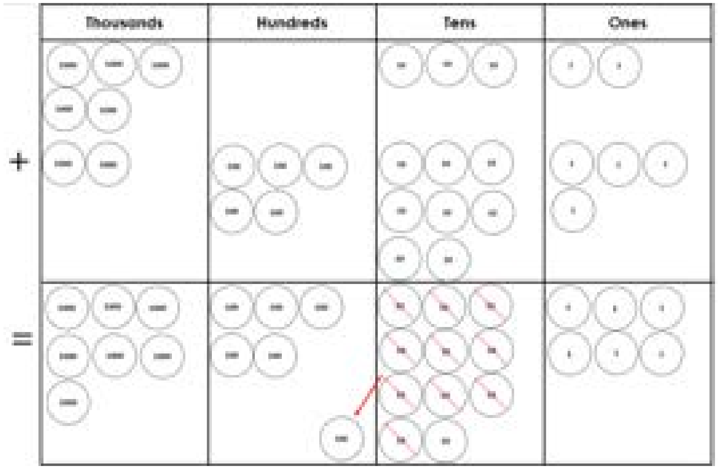
Pictorial

Pupils to draw counters crossing out for regrouping.

3231 + 1322 = 4553



5032 + 2584 = 7616



Abstract

Use of column method to add up to two 4-digit numbers (begin without regrouping and progress to regrouping).

$$\begin{array}{cccc} & 3 & 2 & 3 & 1 \\ + & 1 & 3 & 2 & 2 \\ \hline & 4 & 5 & 5 & 3 \\ \hline \end{array}$$

$$\begin{array}{ccccc} & 5 & 0 & 3 & 2 \\ + & 2 & 5 & 8 & 4 \\ \hline & 7 & 6 & 1 & 6 \\ \hline & & & 1 & \end{array}$$

Lingfield Education Trust

Year 5

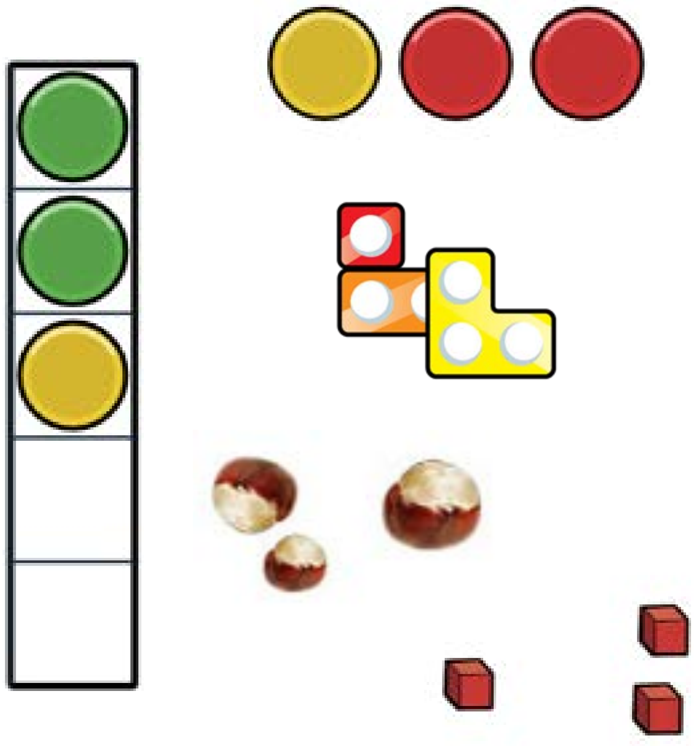
Concrete	Pictorial	Abstract
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.	By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.	Use of column addition for numbers including millions before using for numbers with up to three decimals places. 3,495,032 + 642,584 = $\begin{array}{r} 3\;4\;9\;5\;0\;3\;2 \\ +\;0\;6\;4\;2\;5\;8\;4 \\ \hline 4\;1\;3\;7\;6\;1\;6 \\ \hline {}^{1}\quad\;\;{}^{1}\quad \end{array}$ 341.924 + 64.294 = $\begin{array}{r} 3\;4\;1\,.\,9\;2\;4 \\ +\;\;\;6\;4\,.\,2\;9\;4 \\ \hline 4\;0\;6\,.\,2\;1\;8 \\ \hline {}^{1}\;\;{}^{1}\;\;{}^{1}\quad \end{array}$

National Curriculum
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Year 6

Concrete	Pictorial	Abstract
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.	By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.	Use of column addition for numbers including millions before using for numbers with up to three decimals places. 3,495,032 + 642,584 = 3 4 9 5 0 3 2 + 0 6 4 2 5 8 4 4 1 3 7 6 1 6 1 1 341.924 + 64.294 = 3 4 1 . 9 2 4 + 6 4 . 2 9 4 4 0 6 . 2 1 8 1 1 1

Subtraction

Nursery		
Concrete	**Pictorial**	**Abstract**
Pupils to use a range of practical resources to subtract numbers up to three. 	All subtraction work will fall within the concrete phase with practical resources at this age.	All subtraction work will fall within the concrete phase with practical resources at this age.

There is to be **no** reference to equations in EYFS – neither verbal or written.

ELG
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Reception

Concrete

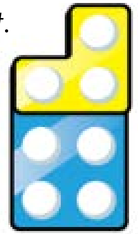
Pupils to use a range of practical resources to understand number decomposition to ten. Ensure this includes numicon overlays for numbers within numbers.

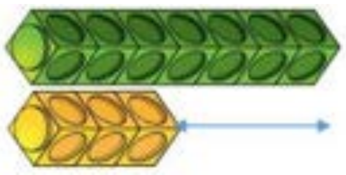
Pupils to use practical resources to understands parts & wholes and numbers inside other numbers for subtraction facts.

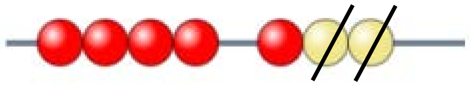
If I have seven and take away a three, four is left.

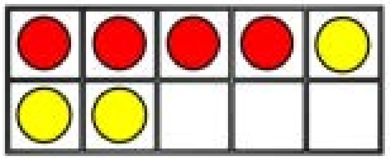
Seven is the whole; four is a part; three is a part.

7 can have a four and a three inside it.



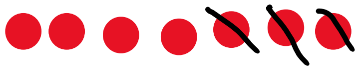


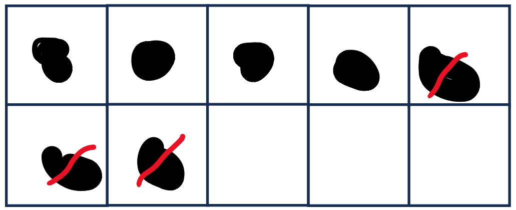


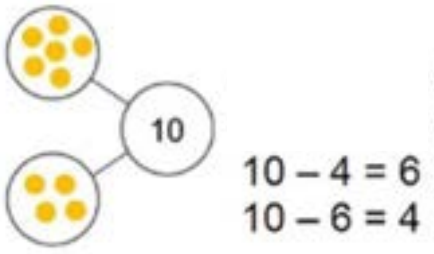


Pictorial

Pupils use simple diagrams, including mark making on prepared ten frames to calculate subtraction statements (number sentences).







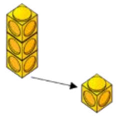
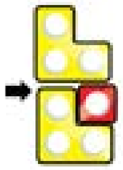
Abstract

All subtraction work will fall within the concrete and pictorial phases with practical resources at this age.

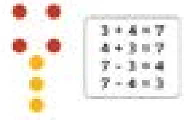
There is to be **no** reference to equations in EYFS – neither verbal or written.

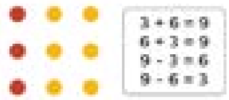
Year 1 (to and within 10)

Concrete	Pictorial	Abstract
Children use a range of concrete resources – **not** including graded number lines – to practice the following subtraction strategies for facts to and within 10 and for those between 10 and 20.	Children represent concrete resources & strategies pictorially.	Pupils to record their subtraction calculations as mathematical statements (number sentences) using the subtraction symbol.

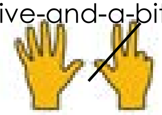
- Subtracting zero
- 1 less 
- 2 less 
- Halves 
- Near halves 7 – 4 = 
- 7-Tree

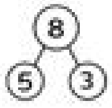
re





- Five-and-a-bit





$$7 - 4 = 3$$

There is to be **no** use of graded number lines for subtraction.

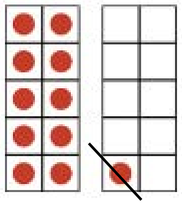
National Curriculum
subtract one-digit and two-digit numbers to 20, including zero; read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs

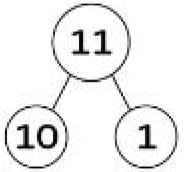
Year 1 (10 to 20)

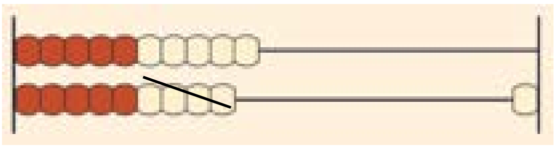
Concrete

Children use a range of concrete resources – **not** including graded number lines – to practice the following subtraction strategies for facts from 10 to 20 (no bridging 10).

- Ten and a bit
- A bit and ten







Pictorial

Children represent concrete resources & strategies pictorially.

Abstract

Pupils to record their subtraction calculations as mathematical statements (number sentences) using the subtraction symbol.

$$17 – 4 = 3$$

There is to be **no** use of graded number lines for subtraction.

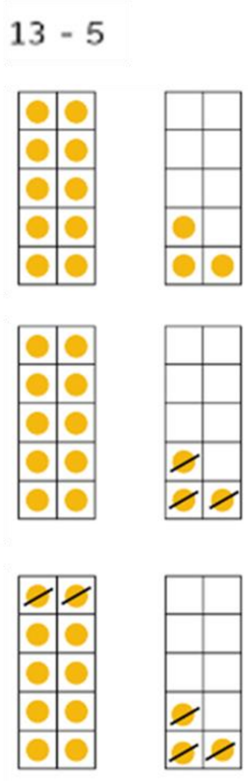
National Curriculum
subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers adding three one-digit numbers

Year 2 (bridging 10)

Concrete

Children use a range of concrete resources – **not** including graded number lines – to practice the following subtraction strategies for bridging ten.

- Make ten and then

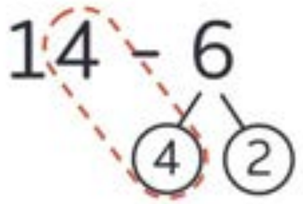


Pictorial

Children represent concrete resources & strategies pictorially.

Abstract

Pupils to use make ten and then in written form.



So,
$14 - 6 = 14 - 4 - 2$
$= 8$

There is to be **no** use of graded number lines for addition.

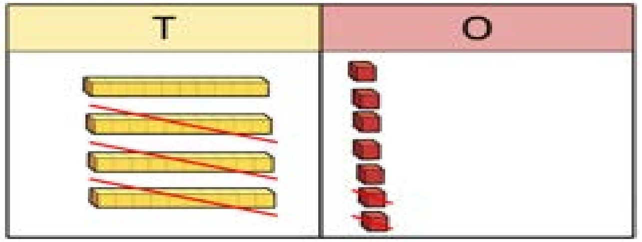
National Curriculum
subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers adding three one-digit numbers

Year 2

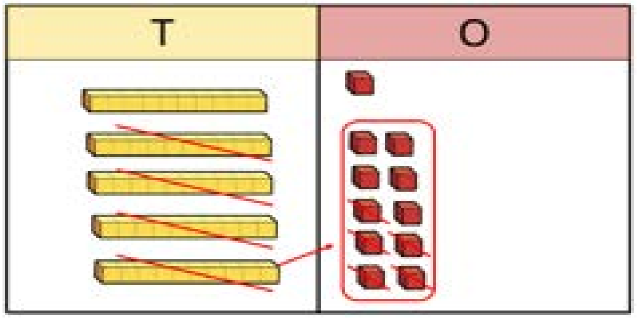
Concrete

Pupils to use Base 10 to practically experience subtracting and regrouping. This must also be done with counters ready for Year 3.

47 - 32 = 15



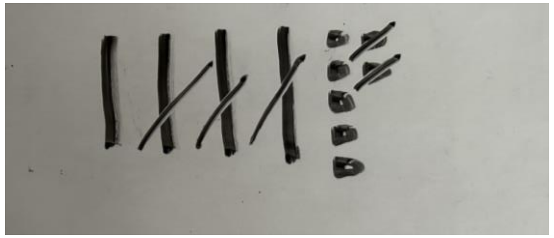
51 - 35 = 16



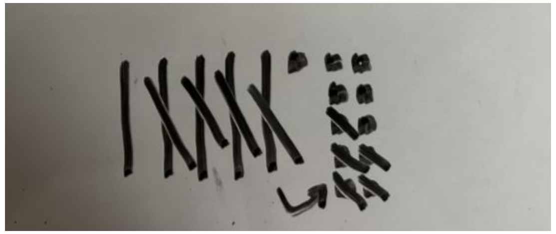
Pictorial

Pupils to draw Base 10 images, which again must move into using counters ready for Year 3.

47 - 32 = 15



51 - 35 = 16



Abstract

Pupils to use expanded method with no regrouping before moving onto it.

47 - 32 = 15

$$\begin{array}{rcccl} & 40 & + & 7 & \\ - & 30 & + & 2 & \\ \hline & 10 & + & 5 & = \quad 15 \\ \hline \end{array}$$

51 - 35 = 16

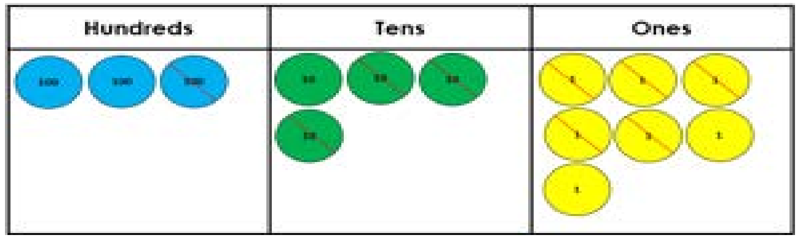
$$\begin{array}{rcccl} & {}^{40}\not{50} & + & {}^{11}\not{1} & \\ - & 30 & + & 5 & \\ \hline & 10 & + & 6 & = \quad 16 \\ \hline \end{array}$$

Year 3

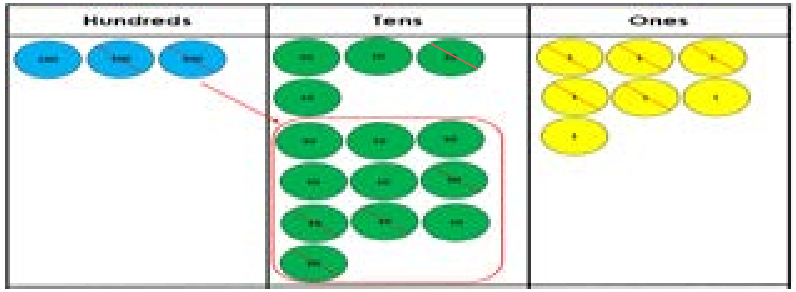
Concrete

Pupils to use Base 10 or counters to practically experience subtracting.

347 – 135 = 212



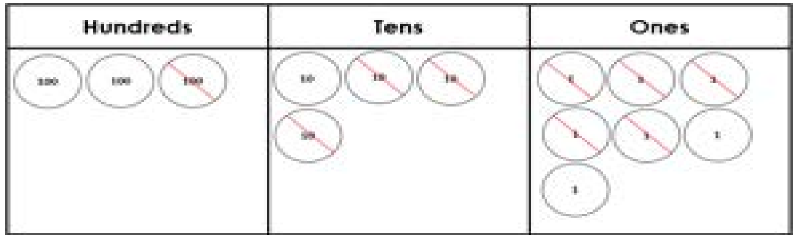
347 – 155 = 192



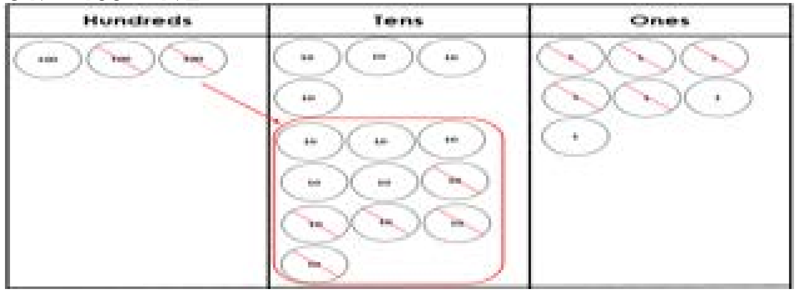
Pictorial

Pupils to draw Base 10 or counters with no regrouping before moving onto it via crossing out.

347 – 135 = 212



347 – 155 = 192



Abstract

Pupils to use expanded method with no regrouping before moving onto it.

	300	+	40	+	7			
-	100	+	30	+	5			
	200	+	10	+	2	=	212	

	~~300~~ 200	+	~~40~~ 140	+	7		
-	100	+	50	+	5		
	100	+	90	+	2	=	192

Pupils to move onto trying compact, standard method ready for Year 4 when secure with the above. **There is no need to do CP stages between the two abstract models;** make sure both abstract models are related to each other in teaching.

	3	4	4
-	2	1	3
	1	3	1

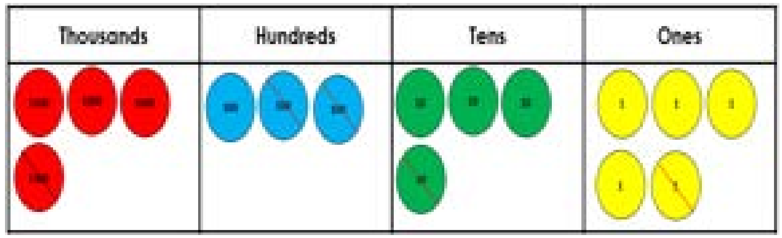
National Curriculum
subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate

Year 4

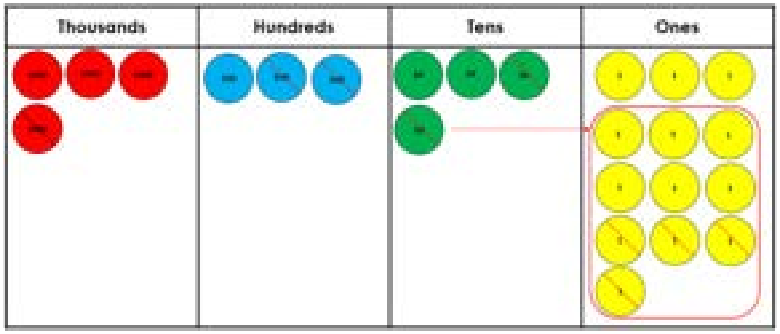
Concrete

Pupils to use counters to practically experience subtracting.

4345 – 1212 = 3133



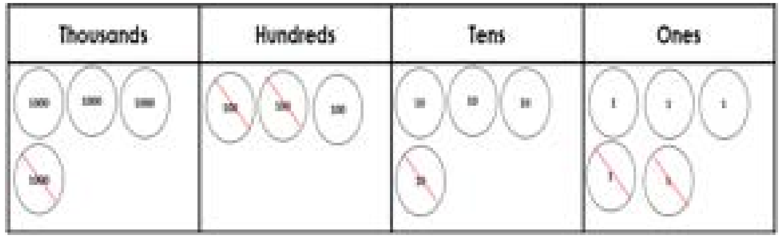
4343 – 1214 = 3129



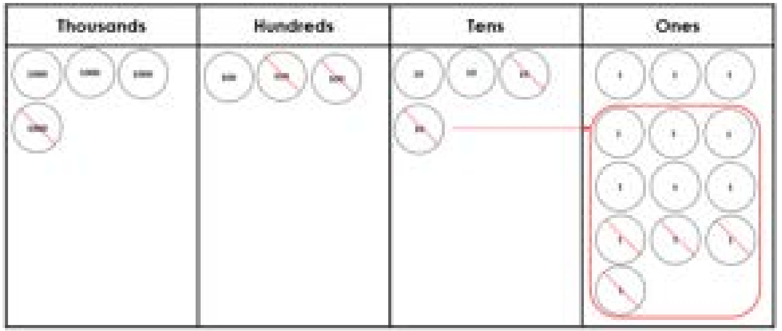
Pictorial

Pupils to draw counters crossing out for regrouping.

4345 – 1212 = 3133



4343 – 1214 = 3129



Abstract

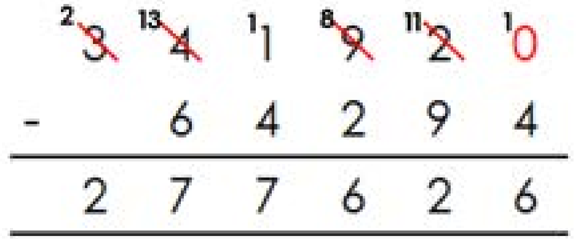
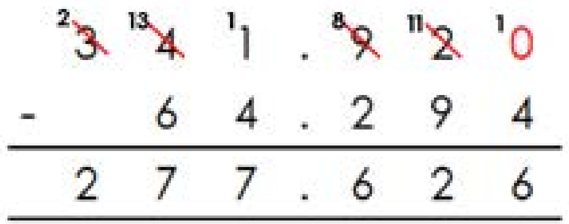
Use of column method to subtract up to two 4-digit numbers (begin without regrouping and progress to regrouping).

$$\begin{array}{rcccc} & 4 & 3 & 4 & 5 \\ - & 1 & 2 & 1 & 2 \\ \hline & 3 & 1 & 3 & 3 \\ \hline \end{array}$$

$$\begin{array}{rcccc} & 4 & 3 & {}^{3}\not{4} & {}^{1}3 \\ - & 1 & 2 & 1 & 4 \\ \hline & 3 & 1 & 2 & 9 \\ \hline \end{array}$$

National Curriculum
subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)

Year 5

Concrete	Pictorial	Abstract
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.	By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.	Use of column subtraction for numbers including millions before using for numbers with up to three decimals places.  

National Curriculum
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Year 6

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use of column subtraction for numbers including millions before using for numbers with up to three decimals places.

$$\begin{array}{rrrrrrr} & {}^{2}\cancel{3} & {}^{13}\cancel{4} & {}^{1}1 & {}^{8}\cancel{9} & {}^{11}\cancel{2} & {}^{1}0 \\ - & & 6 & 4 & 2 & 9 & 4 \\ \hline & 2 & 7 & 7 & 6 & 2 & 6 \\ \hline \end{array}$$

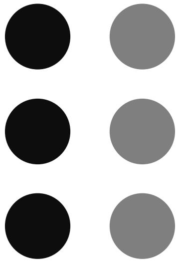
$$\begin{array}{rrrrrrrr} & {}^{2}\cancel{3} & {}^{13}\cancel{4} & {}^{1}1 & . & {}^{8}\cancel{9} & {}^{11}\cancel{2} & {}^{1}0 \\ - & & 6 & 4 & . & 2 & 9 & 4 \\ \hline & 2 & 7 & 7 & . & 6 & 2 & 6 \\ \hline \end{array}$$

Multiplication

Reception

Concrete

Children use physical resources to solve multiplication problems involving doubling.



3 and 3 are parts of 6

3 and 3 are numbers inside 6

Double 3 is 6

Avoid representations which will be remembered more than the mathematical content!

Pictorial

Children use pictorial representations to represent the concrete resources used.

Abstract

Children to work in concrete and pictorial phases only.

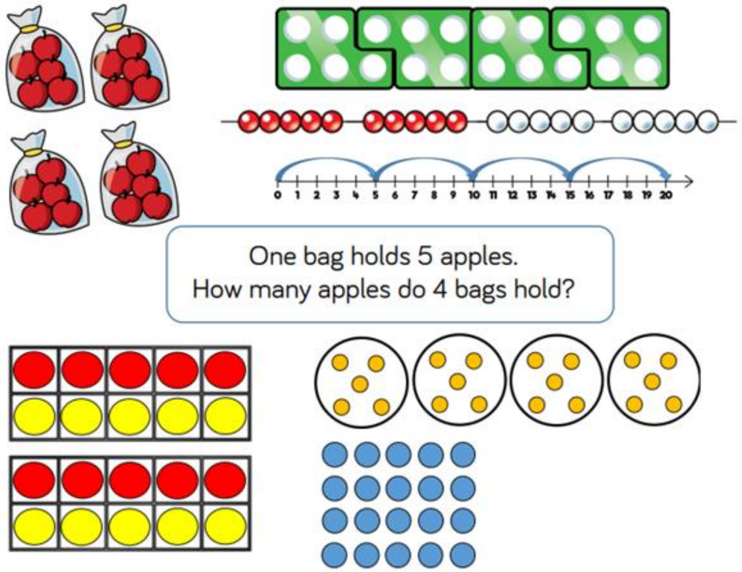
There is to be **no** reference to equations in EYFS – neither verbal or written.

National Curriculum
solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 1

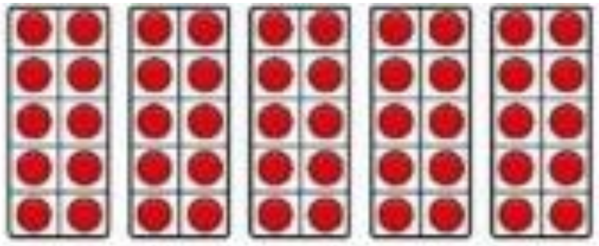
Concrete

In Year 1, children use concrete resources to solve multiplication problems. Children represent multiplication as repeated addition in many different ways. This should include physical, labelled number tracks ready for the pictorial phase.

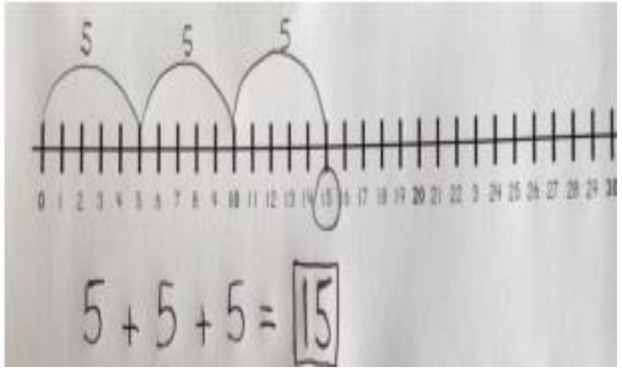


Pictorial

Use pictorial arrays to build understanding of multiplication through counting the total in amounts in 2s, 10s and 5s..



Use a number line to jump in multiples of 2, 5 and 10 (repeated addition).



Abstract

Use mathematical statements (number sentences) for repeated addition of 2, 5 or 10.

$$5 + 5 + 5 = 15$$

Introduce the multiplication symbol to replace repeated addition.

$$5 + 5 + 5 = 15$$

$$3 \times 5 = 15$$

National Curriculum
calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs

Year 2

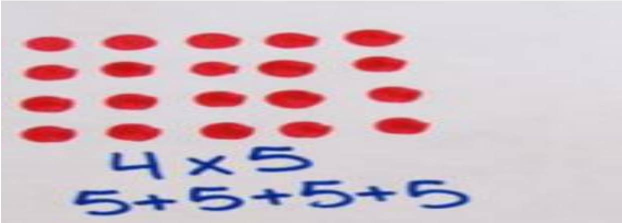
Concrete

Use a range of physical resources to practically experience repeated addition and multiplication.

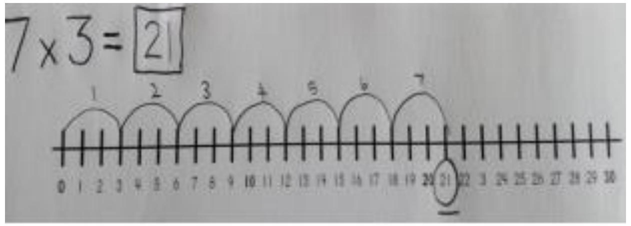
Numicon number tracks used alongside cuisenaire rods are an excellent way to bridge towards the use of number lines for repeated addition.

Pictorial

Make marks to create arrays show repeated addition of 2, 3, 5 or 10.



Use a number line to represent jumps in groups of 2, 3, 5 and 10 (counting on using repeated addition) where the number of jumps will equal the number of groups.



Children can progress to drawing their own number lines.

Abstract

Write repeated addition sentences to match sets of objects or pictures.

$$5 + 5 + 5 + 5 = 20$$

Use the multiplication symbol to replace repeated addition.

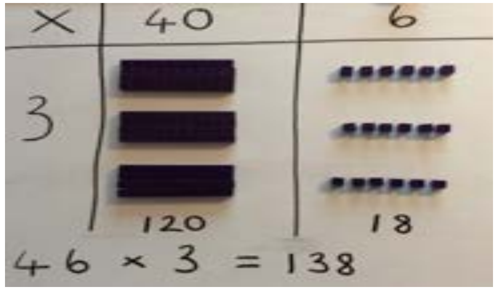
$$7 \times 3 = 21$$

National Curriculum
write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Year 3

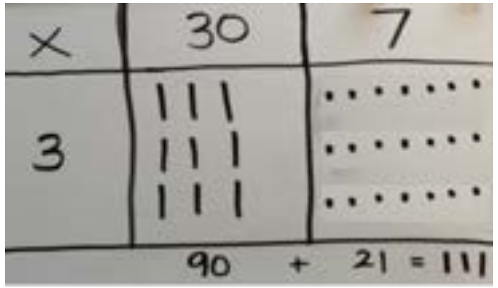
Concrete

Create arrays using dienes and position these correctly on a grid (to introduce the grid method for 2-digit x 1-digit). Progress to use counters ready for Year 4.



Pictorial

Use marks to represent Base10 on a multiplication grid method (2-digit x 1-digit) and likewise for counters.



Abstract

Replace resources/marks with digits on an expanded grid.

x	20	9
	20	9
	20	9
6	20	9
	20	9
	20	9
	20	9

120 + 54 = 174

Move on to the grid method.

x	20	9
6	120	54

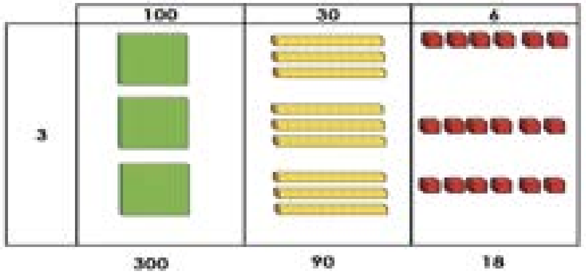
120 + 54 = 174

National Curriculum
multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Year 4

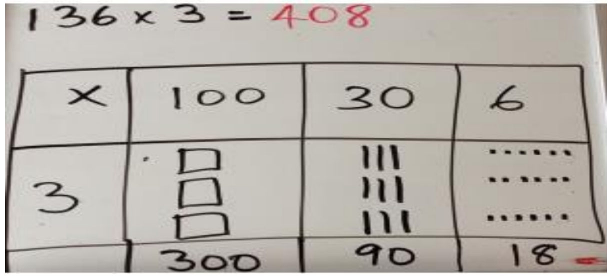
Concrete

Pupils to use Base 10 to support multiplication.



Pictorial

Pupils to create pictorial representations of Base 10 to support multiplication.



Abstract

Introduce short multiplication as a formal written method for multiplying 2 or 3 digit numbers by 1 digit numbers using the expanded method to show the addition of two products.

```
      324
 x      2
 ________
        8      2 x 4
       40      2 x 20
      600      2 x 300
 ________
      648
```

Progress to short multiplication for multiplying 3-digit by 1-digit numbers.

```
      4  5
 x       6
 _________
   2  7  0
 _________
      3
```

```
     3  4  5
 x         6
 ___________
  2  0  7  0
 ___________
     2  3
```

Year 5

Concrete	Pictorial	Abstract
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.	By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.	Use of short multiplication for multiplying 4-digit by 1-digit numbers. $\begin{array}{r} 3\;2\;2\;5 \\ \times \quad\quad\; 4 \\ \hline 1\;2\;9\;0\;0 \\ \hline {}_{1\;2}\;\; \end{array}$ Use of long multiplication for multiplying 4-digit by 2-digit numbers. $\begin{array}{r} 1\;2\;3\;5 \\ \times \quad\quad 2\;1 \\ \hline 1\;2\;3\;5 \\ 2\;4\;7\;0\;0 \\ \hline 2\;5\;9\;3\;5 \\ \hline \end{array}$

National Curriculum
multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Year 6

Concrete	Pictorial	Abstract
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.	By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.	Consolidate use of short and long multiplication with integers from Year 5, before using for decimals with up to two decimal places.

$$\begin{array}{r} 32.25 \\ \times \quad\quad 4 \\ \hline 129.00 \\ \hline {}^{1}\;{}^{2}\quad \end{array}$$

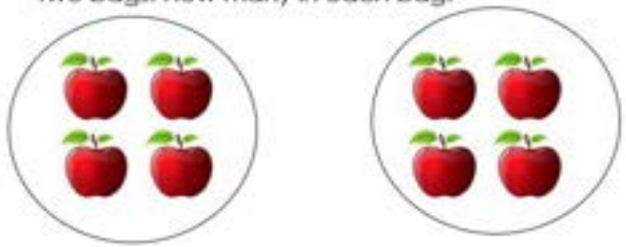
$$\begin{array}{r} 123.5 \\ \times \quad\; 21 \\ \hline 123.5 \\ 24{}^{1}70.0 \\ \hline 2593.5 \\ \hline \end{array}$$

Division

Reception

Concrete	Pictorial	Abstract
Children solve division problems by **sharing** amounts into two equal groups to develop concept of halving. Children use concrete resources to solve problems.	Children use pictorial representations to represent the concrete resources used.	Children to work in concrete and pictorial phases only.

There are eight apples shared equally between two bags. How many in each bag.



Children also solve problems by **grouping** and counting the number of groups.

Put these socks in pairs.

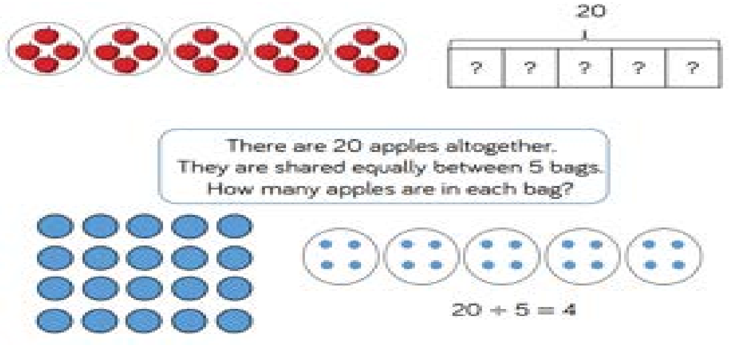
There is to be **no** reference to equations in EYFS – neither verbal or written.

National Curriculum
solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

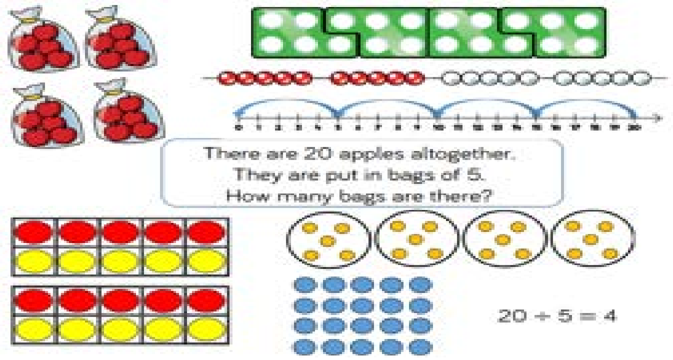
Year 1

Concrete

Children solve division problems by **sharing** amounts into equal groups. Children use concrete resources to solve problems.



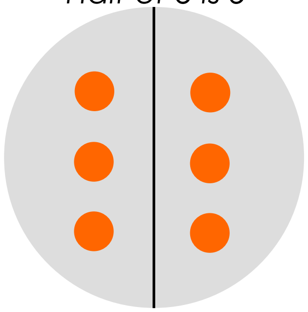
Children also solve problems by **grouping** and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line ready for Year 2.

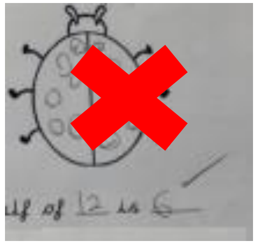


Pictorial

Children solve division problems by sharing amounts into equal groups. Children use pictorial representations to solve problems.

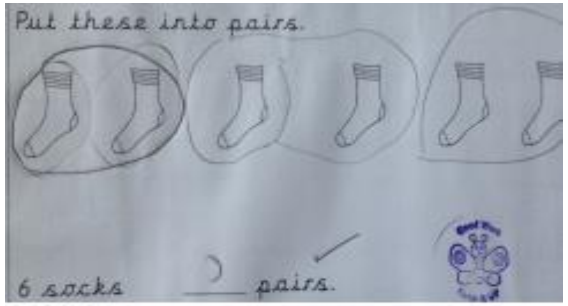
Half of 6 is 3





Avoid representations which will be remembered more than the mathematical content!

Children also solve problems by **grouping** and counting the number of groups using pictorial representations, including number lines ready for Year 2.



Abstract

Introduce the division symbol to record sharing calculations.

$$20 \div 5 = 4$$

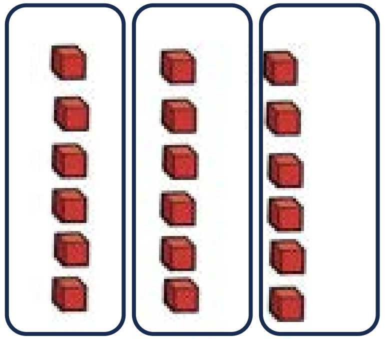
National Curriculum
calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs

Year 2

Concrete

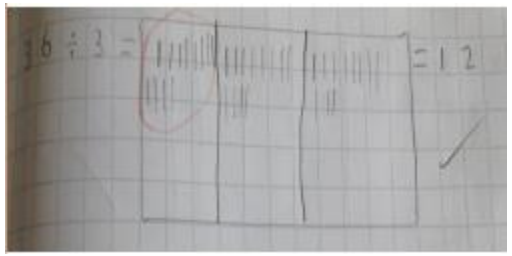
Use of concrete apparatus for sharing and grouping to continue.

18 ÷ 3 = 6

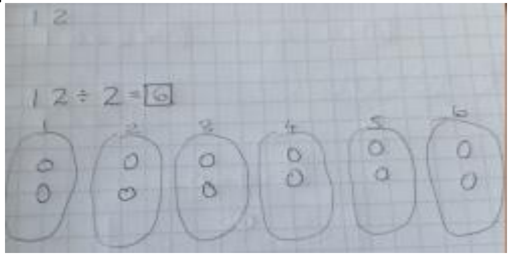


Pictorial

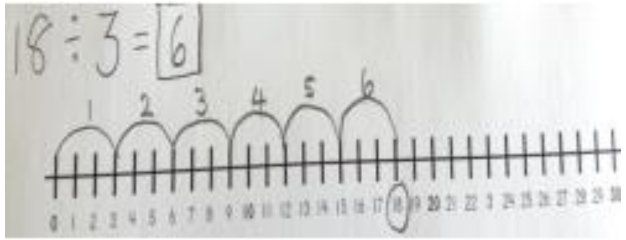
Children make marks to show sharing between 2, 3, 5, or 10.



Children make marks to show division by grouping sets of 2, 3, 5, or 10.



Progress to use of a number line to represent jumps in groups of 2, 3, 5 and 10 (counting on using repeated addition) where the number of jumps will equal the number of groups.



Abstract

Pupils to write their own division statements to record their calculations using the division and equals symbols..

$$18 \div 3 = 6$$

National Curriculum
write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers by one-digit numbers, using mental and progressing to formal written methods

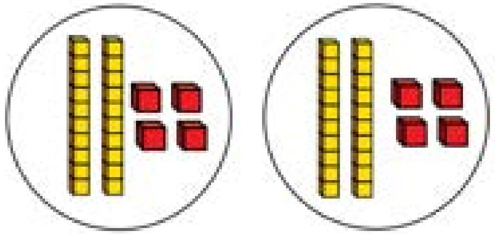
Year 3

Concrete

Use of concrete apparatus for sharing and grouping to continue.

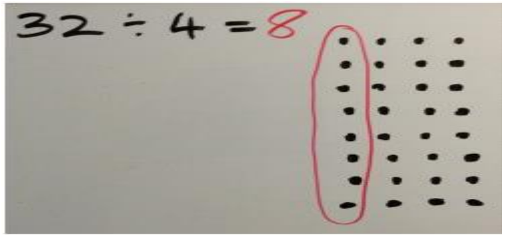
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

48 ÷ 2

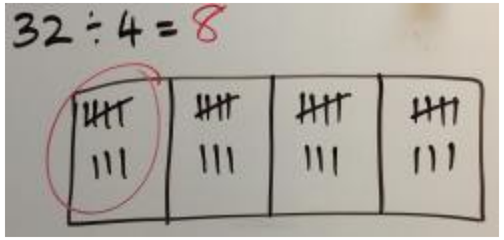


Pictorial

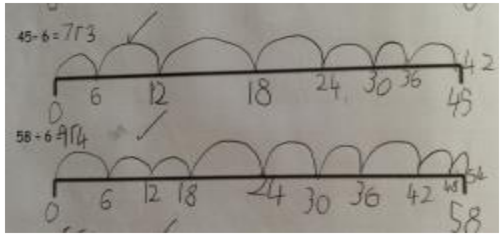
Pupils use marks to show grouping in 4s, 6s and 8s.



Use marks to show sharing in 4s, 6s and 8s.



Use a number line to represent jumps in groups of 2, 3, 4, 5, 6, 8 and 10 (counting on using repeated addition) where the number of jumps will equal the number of groups and the number left over is the remainder.



Abstract

Use the division symbol to record calculations when dividing by 2, 3, 4, 5, 6, 8 and 10. Make explicit links between multiplication and division.

$36 \div 3 = 12$

$36 \div 12 = 3$

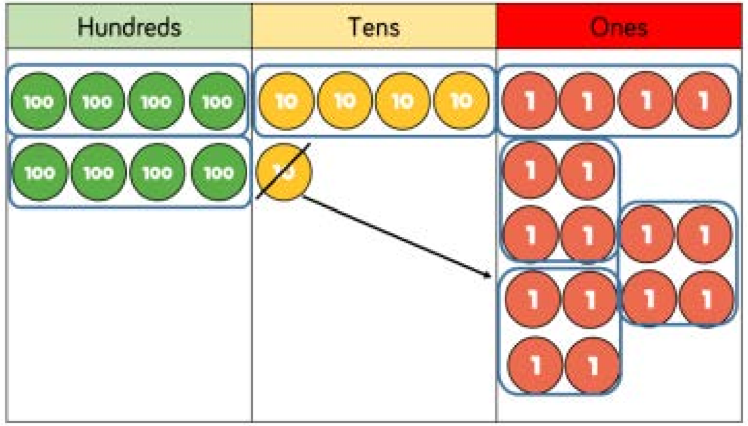
National Curriculum
Pupils practise to become fluent in the formal written method of short division with exact answers

Year 4

Concrete

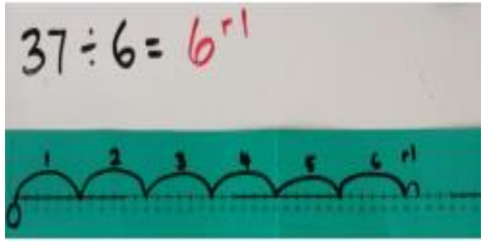
Children can continue to use grouping to support their understanding of short division when dividing a 2 or 3-digit number by a 1-digit number.

856 ÷ 4 = 214

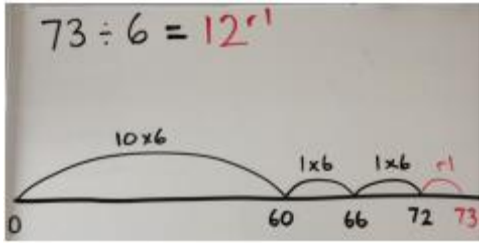


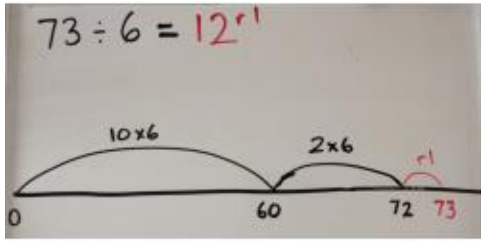
Pictorial

Use a number line to represent jumps in equal groups using all multiplication facts (as in year 3 – repeated addition) if required. This is consolidation and linking to Year 3.



Use a number line to count 'ten lots of' / 'ten groups of' and find remainders (chunking method). Progress to children choosing their own way of chunking using known multiplication facts.





Abstract

Do not use the flexible strategy in White Rose for main written method – that can be used as a mental strategy.

Use of short division for dividing 2-digit and 3-digit numbers by 1-digit numbers (links to the number line work) with no remainders and then remainders.

Start by dividing 2-digit numbers by a 1-digit number with no regrouping or remainder.

$$\begin{array}{r} 1\;\;2 \\ 4\overline{)\,4\;\;8} \end{array}$$

Progress to dividing 3-digit numbers by a 1-digit number with no regrouping or remainder.

$$\begin{array}{r} 1\;\;2\;\;2 \\ 4\overline{)\,4\;\;8\;\;8} \end{array}$$

Move on to dividing 3-digit numbers by a 1-digit number with a remainder but no regrouping within.

$$\begin{array}{r} 1\;\;2\;\;2\text{ r1} \\ 4\overline{)\,4\;\;8\;\;9\;\;\;\;} \end{array}$$

Finish on to dividing 3-digit numbers by a 1-digit number with a remainder and grouping within.

$$\begin{array}{r} 1\;\;4\;\;7\text{ r1} \\ 4\overline{)\,5\;\;{}^{1}8\;\;{}^{2}9\;\;\;\;} \end{array}$$

National Curriculum
divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Year 5

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use short division for up to 4-digit numbers divided by a single digit including remainders .

Start with no remainder and no regrouping within.

    1 2 2 1
4 | 4 8 8 4

Progress to no regrouping within but remainder at the end.

    1 2 2 1 r1
4 | 4 8 8 5

Move to regrouping within and remainder.

    1 2 4 6 r1
4 | 4 9 ^{1}8 ^{2}5

National Curriculum
divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Year 6

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Children need to be taught how to express remainders as decimals via short division in Year 6.

$$\begin{array}{r|l} & 1\ 2\ 4\ 6 \cdot 2\ 5 \\ \hline 4 & 4\ 9\ {}^{1}8\ {}^{2}5 \cdot {}^{1}0\ {}^{2}0 \end{array}$$

National Curriculum
divide numbers up to 4 digits by a two-digit whole number using the formal written method of long (or short) division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate

Year 6

Concrete

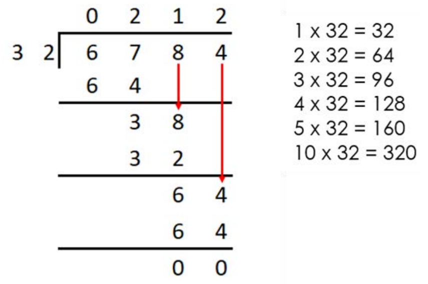
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

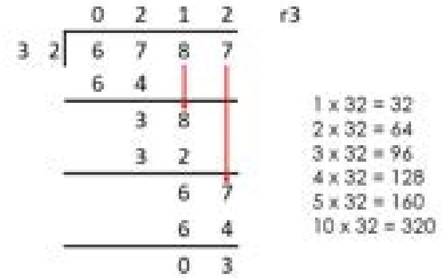
By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

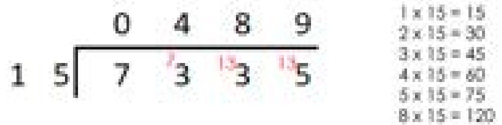
Consolidate short division from Year 5 before introducing long division for 4-digit numbers by 2-digit numbers (you may wish to start with 3-digit by 2-digit).



Move onto dividing 4-digit by 2-digit with remainders.



Look at efficiency and when short division can be used for 4-digit by 2-digit.

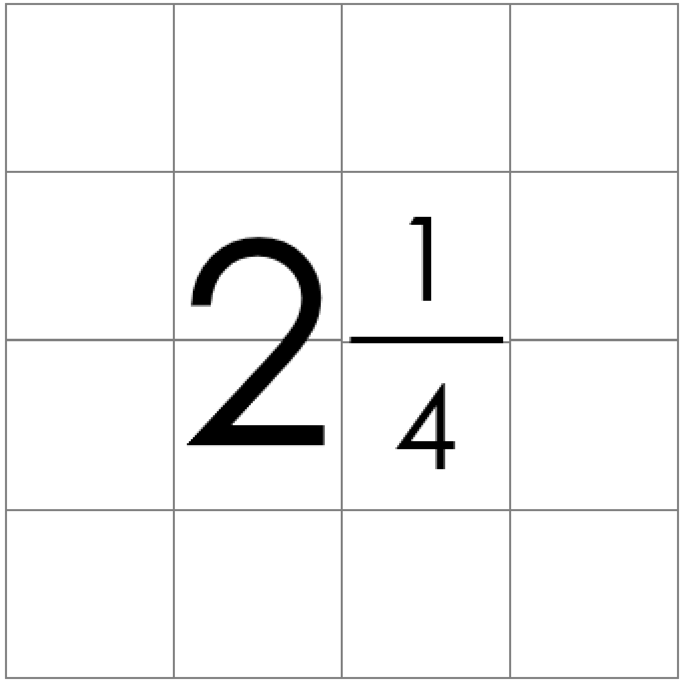


Fractions

Presentation of Fraction Notation

Whole numbers: vertically over two squares

Fraction: one digit per square with vinculum drawn horizontally between the numerator and denominator.



Guidance Boxes

Deal with the wholes first and **avoid needless converting to improper fractions**

Some subtracting mixed number questions require breaking the whole using improper fractions – but only some. It is inefficient to ask children to use this process for addition and other subtraction questions just so they know the process for when it is needed – teach children the most efficient method for the question. This allows rich reasoning discussions about strategy choice.
Avoid needless converting to improper fractions.

Use the language of lowest common denominator and **avoid any use of cross multiplying.**

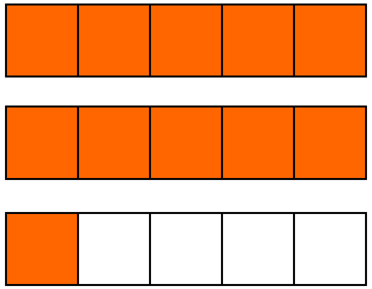
To find common denominators there, is an inefficient strategy known as cross multiplying where you multiply denominators to find common denominators. For example $^{3}/_{8}$ and $^{5}/_{6}$ have a common denominator of 48, however the lowest common denominator is 24. Using the smaller one is better maths (build links) but also more efficient as it leads to simpler, smaller calculations.
Avoid all use of cross multiplying.

Converting Between Improper Fractions and Mixed Numbers

The most effective strategy for children to understand the concepts and structures behind converting between improper fractions and mixed numbers is bar modelling.

Improper Fraction to Mixed Number

What is $^{11}/_{5}$ expressed as a mixed number?



1 whole

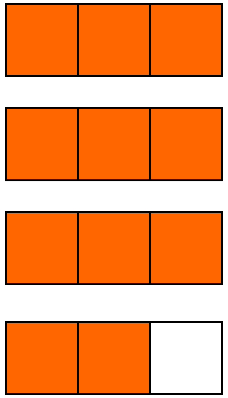
1 whole

$^{1}/_{5}$

$^{11}/_{5} = 2\ ^{1}/_{5}$

Mixed Number to Improper Fraction

What is $3\ ^{2}/_{3}$ expressed as an improper fraction?



$^{3}/_{3}$

$^{3}/_{3}$

$^{3}/_{3}$

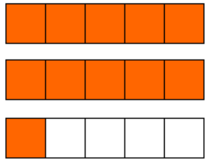
$^{2}/_{3}$

$3\ ^{2}/_{3} = ^{11}/_{3}$

By Year UKS2 , children who are secure with these pictorial methods, should progress to the formal, algorithm.

What is $^{11}/_{5}$ expressed as a mixed number?

$11 \div 5 = 2 \text{ r } 1 = 2^{1}/_{5}$



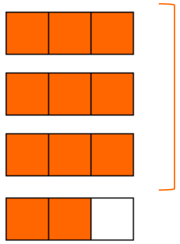
Represent 11 in fifths to match 11 ÷ 5

It left two wholes and 1 extra

What is $3\ ^{2}/_{3}$ expressed as an improper fraction?

$3 \times 3 + 2 = 11 = ^{11}/_{3}$

Again, introduce alongside the bar model at first.



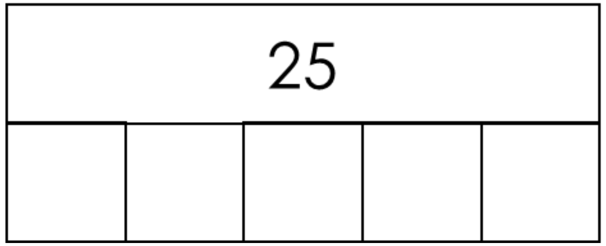
3 lots of 3 = 3 x 3 = 9

2 extra parts to add 9 + 2 = 11

Fractions of Amounts

The most effective strategy for children to understand the concepts and structures behind fractions of amounts is through bar modelling. In this example, the question is what is $^4/_5$ of 25?

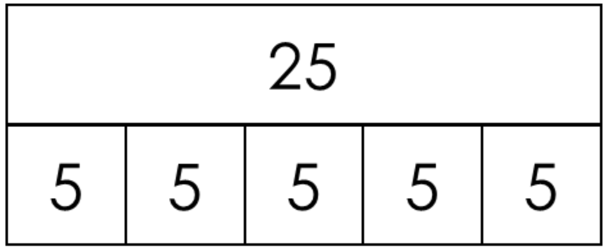
Step 1: represent the amount and denominator as a bar model



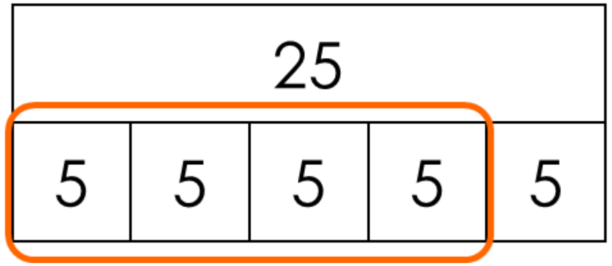
Step 2: represent the bar model as a division equation

25 ÷ 5 = 5

Step 3: populate the parts of the bar model with the quotient



Step 4: mark out the number of parts that are needed to match the numerator.



Step 5: answer the question

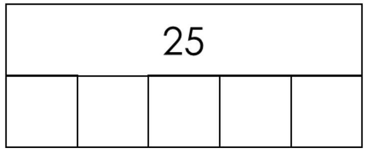
$^4/_5$ of 25 = 4 x 5 = 20

By Year UKS2 , children who are secure with these pictorial methods, should progress to the formal, algorithm. Again, the example is what is $^4/_5$ of 25?

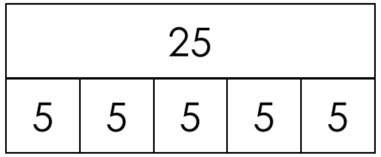
$^4/_5$ of 25 =

25 ÷ 5 x 4 = 20

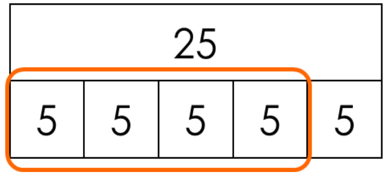
To do this, introduce it alongside the bar model at first.



25 ÷ 5



25 ÷ 5 = 5



5 x 4 = 20

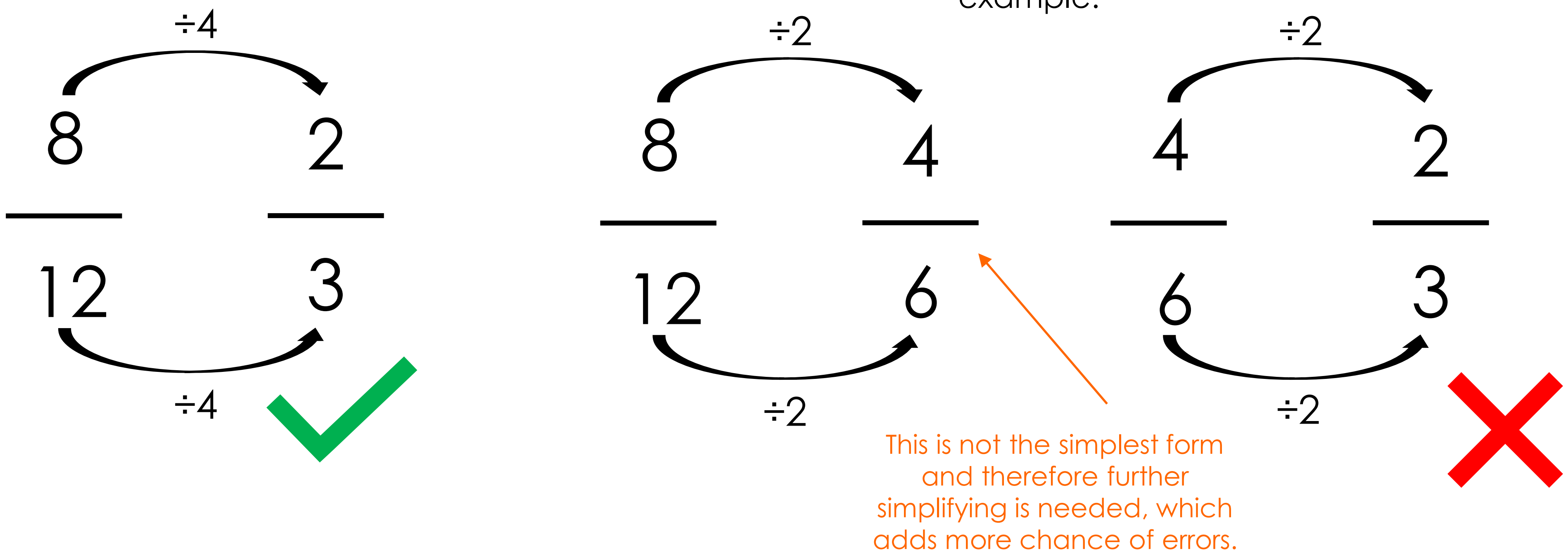
25 ÷ 5 x 4 = 20

Simplifying Fractions

The most effective strategy for children to simplifying fractions is to divide by the highest common factor (HCF). Not using the HCD leads to the need for further simplifying and more chance for errors.

Identify highest common factor (HCF) for division.

Without using HCF children will need to further simplify, for example:



Common Denominators

The most effective strategy for children to find common denominators is to find the lowest common multiple (LCM). Strategies that include cross multiplying (right) are confusing, prone to error, leave larger equivalents to work with and crucially will need unpicking and reteaching by secondary teachers. **The advice of this guidance is that all cross multiplying is poor maths and should be avoided.**

Identify lowest common multiple (LCM) for division.

$$\frac{5}{6} \quad \frac{3}{8}$$

x4 x3

$$\frac{20}{24} \quad \frac{9}{24}$$

✓

Not identifying the lowest common multiple (LCM) for division and cross multiplying (6 x 8) leads to larger equivalents and more chance for error.

$$\frac{5}{6} \times \frac{3}{8}$$

x

$$\frac{40}{48} \quad \frac{18}{48}$$

✗

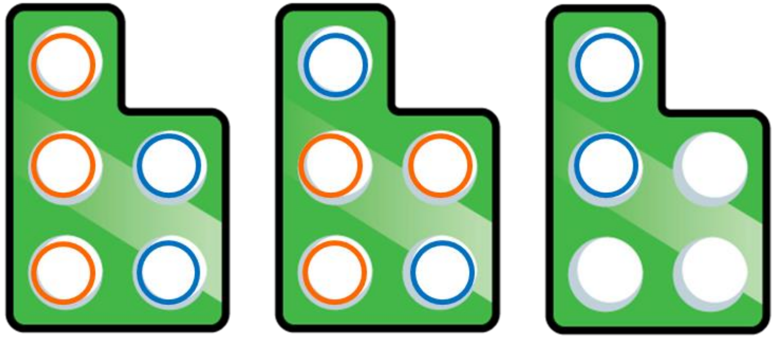
Mathematical Manipulatives

In order for our children to fully understand the structures within calculation involving fractions, we use a set of key maths manipulatives to **expose** the maths.

We expect all of our children to progress to **doing** the maths **without** these resources, when secure.

Numicon

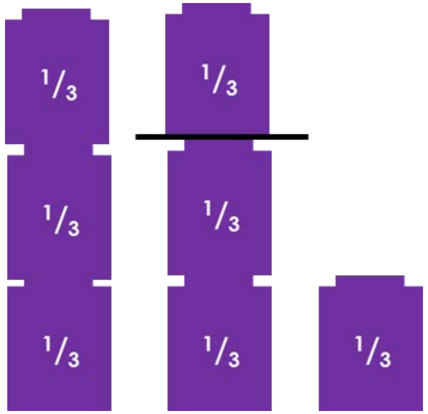
$4 \times {}^{3}/_{5} = 2\,{}^{2}/_{5}$



*Mark/colour alternating holes.

Fraction Cubes/Tower

$1\,{}^{2}/_{3} + {}^{2}/_{3} = 2\,{}^{1}/_{3}$



Fraction Circles

$1 - {}^{1}/_{8} = {}^{7}/_{8}$

Fractions: addition

National Curriculum
add fractions with the same denominator within one whole [for example, $^{5}/_{7} + ^{1}/_{7} = ^{6}/_{7}$]

Year 3

Concrete

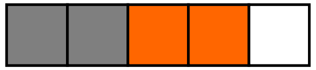
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

Adding fractions with the same denominator within the whole.

$$\frac{2}{5} + \frac{2}{5} =$$



Abstract

Adding fractions with the same denominator within the whole.

$$\frac{2}{5} + \frac{2}{5} = \frac{2+2}{5} = \frac{4}{5}$$

Year 4

Concrete

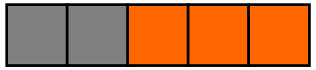
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

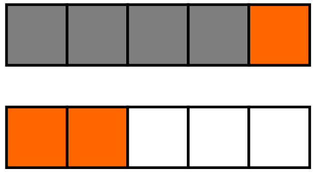
Adding fractions with the same denominator to the whole.

$$\frac{2}{5} + \frac{3}{5} =$$



Adding fractions with the same denominator beyond the whole.

$$\frac{4}{5} + \frac{3}{5} =$$



Abstract

Adding fractions with the same denominator to the whole.

$$\frac{2}{5} + \frac{3}{5} = \frac{2+3}{5} = \frac{5}{5} = 1$$

Adding fractions with the same denominator beyond the whole.

$$\frac{4}{5} + \frac{3}{5} = \frac{4+3}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Year 4

Concrete

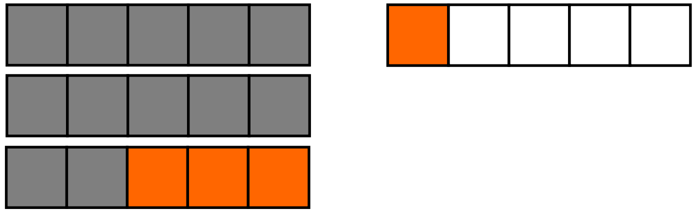
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

Adding a fraction to a mixed number with the same denominator.

$$2\frac{2}{5} + \frac{4}{5} =$$



Abstract

Adding a fraction to a mixed number with the same denominator.

$$2\frac{2}{5} + \frac{4}{5} = 2\frac{2+4}{5} = 2\frac{6}{5} = 3\frac{1}{5}$$

National Curriculum
add fractions with denominators that are multiples of the same number

Year 5

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Adding fractions with denominators from the same multiple family.

Adding fractions with denominators from the same multiple family.

$$\frac{3}{4} + \frac{1}{8}$$

$$\frac{6}{8} + \frac{1}{8} = \frac{6+1}{8} = \frac{7}{8}$$

Maintain the fraction with the larger digit denominator

Use the multiple relationship for fraction with the smaller digit denominator

Use the language of smallest common denominator and **avoid any use of cross multiplying.**

$$\frac{1}{4} + \frac{5}{12}$$

x3

$$\frac{3}{12} + \frac{5}{12} = \frac{3+5}{12} = \frac{8}{12}$$

National Curriculum
add fractions with different denominators using the concept of equivalent fractions

Year 6

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Adding fractions with different denominators not bridging the whole. $\frac{1}{4} + \frac{2}{6}$ (x3, x2) $\frac{3}{12} + \frac{4}{12} = \frac{3+4}{12} = \frac{7}{12}$ Use the language of smallest common denominator and **<u>avoid</u> any use of cross multiplying.**
		Adding fractions with different denominators bridging the whole. $\frac{3}{4} + \frac{5}{6}$ (x3, x2) $\frac{9}{12} + \frac{10}{12} = \frac{9+10}{12} = \frac{19}{12} = 1\frac{7}{12}$ Use the language of smallest common denominator and **<u>avoid</u> any use of cross multiplying.**

Year 6

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Adding a mixed number and fraction with different denominators not bridging the whole. $1\frac{1}{4} + \frac{2}{6}$ (x3, x2) $\frac{3}{12} + \frac{4}{12} = 1\frac{3+4}{12} = 1\frac{7}{12}$ Deal with the wholes first and **<u>avoid</u> needless converting to improper fractions** Use the language of smallest common denominator and **<u>avoid</u> any use of cross multiplying.**
		Adding two mixed numbers with different denominators not bridging the whole. $2\frac{1}{4} + 1\frac{2}{6}$ (x3, x2) $\frac{3}{12} + \frac{4}{12} = 3\frac{3+4}{12} = 3\frac{7}{12}$ Deal with the wholes first and **<u>avoid</u> needless converting to improper fractions** Use the language of smallest common denominator and **<u>avoid</u> any use of cross multiplying.**

Year 6

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Adding a mixed number and fraction with different denominators bridging the whole. $2\frac{3}{4} + \frac{5}{6}$ (x3, x2) $\frac{9}{12} + \frac{10}{12} = 2\frac{9+10}{12} = 2\frac{19}{12} = 3\frac{7}{12}$ Deal with the wholes first and **<u>avoid</u> needless converting to improper fractions** Use the language of lowest common denominator and **<u>avoid</u> any use of cross multiplying.**
		Adding two mixed numbers with different denominators bridging the whole. $2\frac{3}{4} + 1\frac{5}{6}$ (x3, x2) $\frac{9}{12} + \frac{10}{12} = 3\frac{9+10}{12} = 3\frac{19}{12} = 4\frac{7}{12}$ Deal with the wholes first and **<u>avoid</u> needless converting to improper fractions** Use the language of lowest common denominator and **<u>avoid</u> any use of cross multiplying.**

Fractions: subtraction

National Curriculum
subtract fractions with the same denominator within one whole [for example, 5/7 - 1/7 = 4/7]

Year 3

Concrete

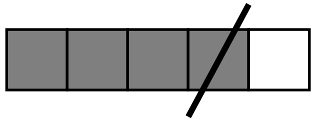
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

Subtracting fractions with the same denominator within the whole.

$$\frac{4}{5} - \frac{1}{5} =$$



Abstract

Subtracting fractions with the same denominator within the whole.

$$\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$$

Year 4

Concrete

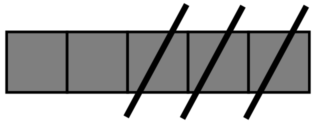
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

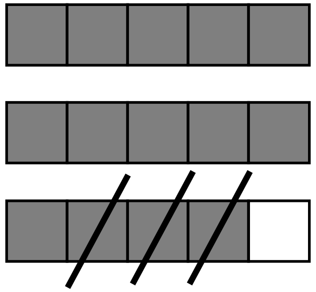
Subtracting fractions with the same denominator within the whole.

$$1 - \frac{3}{5} =$$



Subtracting fractions from mixed numbers – <u>no</u> questions that involve breaking the whole.

$$2\frac{4}{5} - \frac{3}{5} =$$



Abstract

Subtracting fractions with the same denominator within the whole.

$$1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$$

Subtracting fractions from mixed numbers – <u>no</u> questions that involve breaking the whole.

$$2\frac{4}{5} - \frac{3}{5} = 2\frac{4-3}{5} = 2\frac{1}{5}$$

Deal with the wholes first and <u>avoid</u> needless converting to improper fractions

Year 5

Concrete

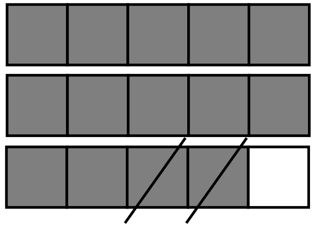
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

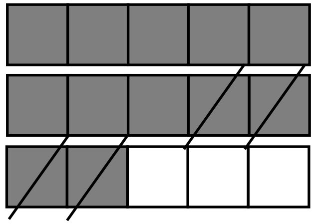
Subtracting a fraction from a mixed number with the same denominator.

$$2\frac{4}{5} - \frac{2}{5} =$$



Subtracting a fraction from a mixed number with the same denominator – breaking the whole.

$$2\frac{2}{5} + \frac{4}{5} =$$



Abstract

Subtracting a fraction from a mixed number with the same denominator – not breaking the whole.

$$2\frac{4}{5} - \frac{2}{5} = 2\frac{4-2}{5} = 2\frac{2}{5}$$

Subtracting a fraction from a mixed number with the same denominator – breaking the whole.

$$2\frac{2}{5} - \frac{4}{5}$$

$$\frac{12}{5} - \frac{4}{5} = \frac{8}{5} = 1\frac{3}{5}$$

National Curriculum
subtract fractions with denominators that are multiples of the same number

Year 5

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Subtracting fractions with denominators from the same multiple family.

Use the language of lowest common denominator and **avoid any use of cross multiplying.**

$$\frac{3}{4} - \frac{3}{8} \quad \xrightarrow{\times 2} \quad \frac{6}{8} - \frac{3}{8} = \frac{6-3}{8} = \frac{3}{8}$$

x2

Use the multiple relationship for the fraction with the smaller digit denominator

Maintain the fraction with the larger digit denominator

$$\frac{7}{8} - \frac{3}{4} \quad \xrightarrow{\times 2} \quad \frac{7}{8} - \frac{6}{8} = \frac{7-6}{8} = \frac{1}{8}$$

x2

Use the multiple relationship for the fraction with the smaller digit denominator

Maintain the fraction with the larger digit denominator

$$\frac{11}{12} - \frac{3}{4} \quad \xrightarrow{\times 3} \quad \frac{11}{12} - \frac{9}{12} = \frac{11-9}{12} = \frac{2}{12}$$

x3

Use the multiple relationship for the fraction with the smaller digit denominator

Maintain the fraction with the larger digit denominator

$$\frac{3}{4} - \frac{7}{12} \quad \xrightarrow{\times 3} \quad \frac{9}{12} - \frac{7}{12} = \frac{9-7}{12} = \frac{2}{12}$$

x3

Use the multiple relationship for the fraction with the smaller digit denominator

Maintain the fraction with the larger digit denominator

Year 6

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Subtracting fractions with different denominators. $\frac{3}{4} - \frac{1}{6}$ (x3, x2) $\frac{9}{12} - \frac{2}{12} = \frac{9 - 2}{12} = \frac{7}{12}$ Use the language of lowest common denominator and **<u>avoid</u> any use of cross multiplying.** $\frac{5}{6} - \frac{3}{8}$ (x4, x3) $\frac{20}{24} - \frac{9}{24} = \frac{20 - 9}{24} = \frac{11}{24}$

National Curriculum
subtract mixed numbers with different denominators
using the concept of equivalent fractions

Year 6

Concrete

Concrete resources do not provide sensible support for this stage.

Pictorial

Concrete resources do not provide sensible support for this stage.

Abstract

Subtracting a fraction from a mixed number with different denominators not breaking the whole.

$$1\frac{3}{4} - \frac{1}{6}$$

x3 x2

$$\frac{9}{12} - \frac{2}{12} = 1\frac{9-2}{12} = 1\frac{7}{12}$$

Deal with the wholes first and **avoid needless converting to improper fractions**

Use the language of smallest common denominator and **avoid any use of cross multiplying.**

Subtracting a mixed number from a mixed number with different denominators not breaking the whole.

$$2\frac{3}{4} - 1\frac{1}{6}$$

Subtract wholes first

x3 x2

$$\frac{9}{12} - \frac{2}{12} = 1\frac{9-2}{12} = 1\frac{7}{12}$$

Deal with the wholes first and **avoid needless converting to improper fractions**

Use the language of smallest common denominator and **avoid any use of cross multiplying.**

Year 6

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	Subtracting a fraction from a mixed number with different denominators breaking the whole. $$2\frac{3}{4} - \frac{5}{6}$$ $$\frac{11}{4} - \frac{5}{6}$$ x3 x2 $$\frac{33}{12} - \frac{10}{12} = \frac{33 - 10}{12} = \frac{23}{12} = 1\frac{11}{12}$$ Use the language of smallest common denominator and **<u>avoid</u> any use of cross multiplying.** Subtracting a mixed number from a mixed number with different denominators breaking the whole. $$4\frac{3}{4} - 2\frac{5}{6}$$ $$\frac{19}{4} - \frac{17}{6}$$ x3 x2 $$\frac{33}{12} - \frac{22}{12} = \frac{33 - 22}{12} = \frac{11}{12}$$ Use the language of smallest common denominator and **<u>avoid</u> any use of cross multiplying.**

Year 6

Concrete	Pictorial	Abstract
Concrete resources do not provide sensible support for this stage.	Concrete resources do not provide sensible support for this stage.	An alternative approach is to decompose the fraction that is the subtrahend to make a simpler calculation. This is effective when multiplying into common denominators can lead to large amounts. $2\frac{3}{21} - 1\frac{7}{21} = 2\frac{3}{21} - 1\frac{3}{21} = 1$ $\frac{7}{21} \rightarrow \frac{3}{21} \quad \frac{4}{21}$ $1 - \frac{4}{21} = \frac{17}{21}$ $3\frac{1}{4} - 1\frac{3}{4}$ $1 \rightarrow \frac{4}{4}$ Add to the mixed number that is the minuend $3\frac{5}{4} - 1\frac{3}{4} = 2\frac{5-3}{4} = 1\frac{2}{4}$

Fractions: multiplication

National Curriculum
multiply mixed numbers by whole numbers, supported by materials and diagrams

Year 5

Concrete

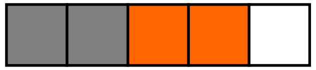
Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Pictorial

Fraction multiplied by whole number not bridging 1 whole.

$2 \times \frac{2}{5} =$



Fraction multiplied by whole number bridging 1 whole.

$4 \times \frac{2}{5} =$

Abstract

Fraction multiplied by whole number not bridging 1 whole.

$$2 \times \frac{2}{5} = \frac{2 \times 2}{5} = \frac{4}{5}$$

Fraction multiplied by whole number bridging 1 whole.

$$4 \times \frac{2}{5} = \frac{4 \times 2}{5} = \frac{8}{5} = 1\frac{3}{5}$$

National Curriculum
multiply proper fractions by whole numbers, supported by materials and diagrams

Year 5

Concrete

Use of the following manipulatives in the same manner as pictorial phase:

- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

When multiplying fractions by large amounts (for example 260 x $^{3}/_{4}$) use fractions of amounts strategies.

Pictorial

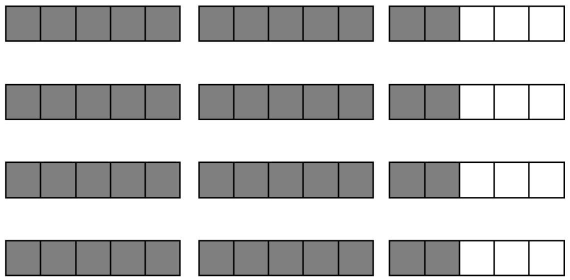
Mixed Number multiplied by whole number <u>not</u> bridging 1 whole.

$$2 \times 2\frac{2}{5} =$$

$= 4\ ^{4}/_{5}$

Mixed Number multiplied by whole number bridging 1 whole.

$$4 \times 2\frac{2}{5} =$$



$$= 8\frac{8}{5} = 9\frac{3}{5}$$

Abstract

Mixed Number multiplied by whole number <u>not</u> bridging 1 whole.

$$2 \times 2\frac{2}{5} =$$

$$4\frac{2 \times 2}{5} = \frac{4}{5} = 4\frac{4}{5}$$

Mixed Number multiplied by whole number bridging 1 whole.

$$4 \times 2\frac{2}{5} = 8\frac{4 \times 2}{5} = 8\frac{8}{5} = 9\frac{3}{5}$$

National Curriculum
multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]

Year 6

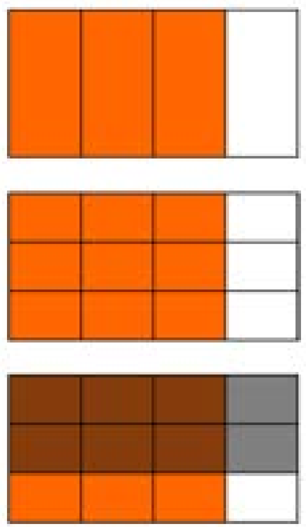
Concrete

Concrete resources do not provide sensible support for this stage.

Pictorial

Proper fraction multiplied by proper fraction.

$$\frac{3}{4} \times \frac{2}{3} =$$



Represent the multiplier vertically.

Draw horizontal lines to represent The multiplicand and colour.

Any parts coloured both ways form the factor.

$$= \frac{6}{12} = \frac{1}{2}$$

Abstract

Proper fraction multiplied by proper fraction.

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$$

Fractions: division

Year 6

Concrete

Use of the following manipulatives in the same manner as pictorial phase:

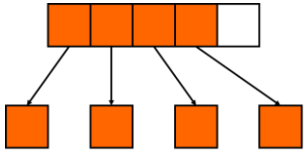
- Fraction cubes/towers
- Fraction Circles
- Numicon (choose the piece matching the denominator and colour alternating holes to match the numerator)

Concrete resources do not provide sensible support for this stage.

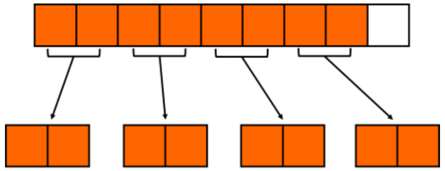
Pictorial

When numerator and divisor match or are from the same multiple family, divide numerator by divisor.

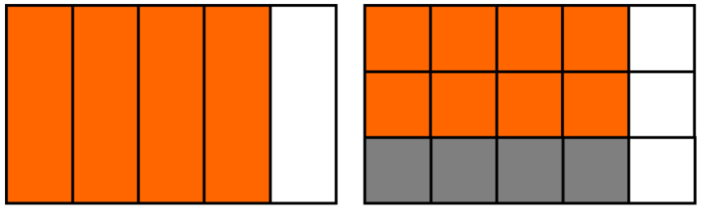
$^4/_5 \div 4$



$^8/_9 \div 4$



When numerator and divisor do not match or are not from the same multiple family, use overlays.



$= {}^4/_{15}$

Represent the fraction vertically.

$$\frac{4}{5} \div 3 =$$

Represent division by the whole horizontally (the whole is how many times to divide it horizontally.

The quotient is one row of the division (if dividing by 3 you would get one row out of 3).

Abstract

When numerator and divisor match or are from the same multiple family, divide numerator by divisor.

$$\frac{4}{5} \div 4 = \frac{4 \div 4}{5} = \frac{1}{5}$$

$$\frac{8}{9} \div 4 = \frac{8 \div 4}{9} = \frac{2}{9}$$

When numerator and divisor do not match or are not from the same multiple family, use formal inverse procedure.

$$\frac{4}{5} \div 3 =$$

$$\frac{4}{5} \div \frac{3}{1} = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

Acknowledgements

Resources Used

- White Rose Calculation Policy
- Captain Cook Primary School Calculation Policy
- NPCAT Calculation Policy
- Purposeful Maths Calculation Policy
- LET EYFS Ready Documents
- Power Maths Calculation Policy

Many thanks to the following contributors
K Yates (Hurworth), P Mayes (Corporation Road), P Foulds (Lingfield Education Trust), C Harrison (Captain Cook), A Poulter (Vane Road), S Guest (Cambrai), R Farrier (Handale), R Carass (Heathfield), S West (Woodham Burn), D Docherty (Marton Manor), Emma Bould (Viewley Hill), Joanne Knox (Hemlington Hall), C Dyer (Mount Pleasant)